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FUZZY IMPLICATIONS AND INFERENCE PROCESSES

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Abstract. We define fuzzy implications in general, then study their families defined from t-norms, t-conorms and strong negations. Connections between such implications and negations are established. Some basic results are presented concerning the contrapositive symmetry property. The study gives birth to a new class of t-norms. Members of this family, together with the corresponding R-implications, have attractive properties making them competitive in different applications, especially in fuzzy inference rules.

Keywords: Fuzzy inference, implications, t-norms and t-conorms, nilpotent minimum

1 INTRODUCTION

Since modeling "if ... then ..." rules with fuzzy predicates is based on fuzzy implications, it is essential to study their mathematical properties. In fuzzy logic, the basic theory of connectives AND, OR, NOT is well-developed and their functional models (t-norms, t-conorms and strong negations) are widely accepted (see e.g. [17, 9, 10]). However, there is no such clear and – in some sense – unique way of defining fuzzy implications.

The present paper intends to give a general definition first, based on [7] and [6]. The main idea behind is simple. If we have a fuzzy implication then we want to consider its reciprocal, also a fuzzy implication. Then, we study implications that are defined by t-norms, t-conorms and strong negations. For such implications, we investigate their relationship with negations, and their contrapositive symmetry. Finally, we recall a family of recently found t-norms, t-conorms and implications. Their attractive properties can be useful also in applications.

2 FUZZY IMPLICATIONS IN GENERAL

As we said, there exist several approaches to the definition of fuzzy implications. The following axioms try to catch their most general and characteristic properties. For justifications and more details we refer to the book by Fodor and Roubens [7].

Definition 1. A *fuzzy implication* is a function $I : [0,1]^2 \to [0,1]$ satisfying the following conditions:

I1. If $x \leq z$ then $I(x,y) \geq I(z,y)$ for all $y \in [0,1]$ [15].

I2. If $y \leq t$ then $I(x,y) \leq I(x,t)$ for all $x \in [0,1]$ [15].

I3. $I(0,y) = 1$ (falsity implies anything) [15].

I4. $I(x,1) = 1$ (anything implies tautology) [7].

I5. $I(1,0) = 0$ (Booleanity) [7].

Suppose N is a strong negation (i.e., a strictly decreasing, continuous function $N : [0,1] \to [0,1]$ with $N(0) = 1$, $N(1) = 0$ and $N(N(x)) = x$ for all $x \in [0,1]$, see [14]) and I is a fuzzy implication. Then the N-*reciprocal* of I is defined by

$$I^N(x,y) := I(N(y), N(x)), \quad x, y \in [0,1]. \tag{1}$$

Clearly, thus defined I^N is also a fuzzy implication.

Now we recall further axioms, in terms of function I. These properties are required in different papers and they could be important also in some applications (see e.g. [2, 3] and further references there).

I6. $I(1,x) = x$ (tautology cannot justify anything) [15].

I7. $I(x, I(y,z)) = I(y, I(x,z))$ (exchange principle) [15].

I8. $x \leq y$ if and only if $I(x,y) = 1$ (implication defines an ordering) [8].

I9. $I(x,0) = N(x)$ is a strong negation [7].

I10. $I(x,y) \geq y$ [19].

I11. $I(x,x) = 1$ (identity principle) [1].

I12. $I(x,y) = I(N(y), N(x))$ with a strong negation N [7].

I13. I is a continuous function [7].

It can be proved (see [18, 7]) that if a function $I : [0,1]^2 \to [0,1]$ fulfils conditions I2, I7, I8 then it satisfies also I1, I3, I4, I5, I6, I10 and I11. This result indicates that properties I9, I12 and I13 are not consequences of the others, and may be essential in obtaining particular families of implications, as we will see later.

3 IMPLICATIONS BASED ON T-NORMS, T-CONORMS AND NEGATIONS

Since t-norms, t-conorms and strong negations are well-accepted models for AND, OR, NOT, respectively, fuzzy implications cannot be studied independently of these operations.

The two most important families of such implications are related either to the formalism of Boolean logic or to a residuation concept from intuitionistic logic. Thus, we introduce the following definitions:

Definition 2. An *S-implication* associated with a t-conorm S and a strong negation N is defined by

$$I_{S,N}(x, y) = S(N(x), y). \tag{2}$$

An *R-implication* associated with a t-norm T is defined by

$$I_T(x, y) = \sup\{z \mid T(x, z) \leq y\}. \tag{3}$$

It is easy to see that both $I_{S,N}$ and I_T satisfy conditions I1–I5 for any t-norm T, t-conorm S and strong negation N, thus they are fuzzy implications. Note also that t-norms and their R-implications satisfying the following residuation condition

$$T(x, z) \leq y \iff I_T(x, y) \geq z, \quad \forall x, y, z \in [0, 1] \tag{4}$$

are especially important (see e.g. [7, 9, 10]). In fact, property (4) is equivalent to left-continuity of T.

For the sake of completeness we mention a third type of implications used in quantum logic and called QL-implication:

$$I_{T,S,N}(x, y) = S(N(x), T(x, y)).$$

In general, $I_{T,S,N}$ violates property I1. Conditions under that I1 is satisfied by a QL-implication can be found in [6].

Now we cite characterization of S-implications (see [15, 7]).

Theorem 1. An implication is an S-implication with an appropriate t-conorm S and a strong negation N if and only if I satisfies I6,I7 and I12.

Characterization of implications that can be defined as R-implications based on left-continuous t-norms (see also [11, 4, 5, 7]) is given as follows:

Theorem 2. A function $I : [0, 1]^2 \to [0, 1]$ is an R-implication based on a left-continuous t-norm if and only if I satisfies conditions I2, I7, I8 and $I(x, .)$ is right-continuous for any fixed $x \in [0, 1]$.

4 NEGATIONS DEFINED BY IMPLICATIONS

Property I9 requires that $N(x) := I(x, 0)$, $x \in [0, 1]$ should be a strong negation. This corresponds to a connection between implications and negations in Boolean logic. It can be proved (see [18, 7]) that if I is a fuzzy implication then the function $I(., 0)$ is a negation in a broad sense (i.e., it is non-increasing and is a Boolean negation). However, it is neither strictly decreasing nor continuous in general.

Proposition 1. Suppose that I is a fuzzy implication. If $n(x) := I(x, 0)$, $x \in [0, 1]$ is continuous then it is involutive and I fulfils I12 with $N(x) := I(x, 0)$, $x \in [0, 1]$.

Note that continuity of the implication is sufficient but not necessary to obtain strong negation via residuation. As an example, consider the particular t-norm (called *nilpotent minimum*, see [6]):

$$\min_0(x, y) := \begin{cases} 0 & \text{if } x + y \leq 1 \\ \min(x, y) & \text{if } x + y > 1. \end{cases}$$

Then its residuated implication is of the form

$$I_{\min_0}(x, y) = \begin{cases} 1 & \text{if } x \leq y \\ \max(1 - x, y) & \text{otherwise.} \end{cases}$$

Although $I_{\min_0}$ is not continuous, $I_{\min_0}(x, 0) = 1 - x$, $x \in [0, 1]$ is the standard strong negation.

When I_T is continuous then we can represent it as a φ-transform of the Łukasiewicz implication. This was proved in [13]. They required more conditions than necessary (see [7]).

Theorem 3. A function $I : [0, 1]^2 \to [0, 1]$ is such that I2, I7, I8 and I13 are satisfied if and only if there exists an automorphism φ of the unit interval such that

$$I(x, y) = \varphi^{-1}(\min\{1 - \varphi(x) + \varphi(y), 1\}). \tag{5}$$

For positive t-norms T (i.e., when $T(x, y) > 0$ for $x, y > 0$) like min or product, the negation obtained via R-implication is not continuous at all:

$$I_T(x, 0) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x > 0. \end{cases}$$

5 CONTRAPOSITIVE SYMMETRY OF FUZZY IMPLICATIONS

In the framework of two-valued logic, a proposition "if P then Q" is true if and only if its contrapositive, "if not-Q then not-P" is true. If I is a fuzzy implication and N is a strong negation, property I12 expresses *contrapositive symmetry* of I with respect to N (CPS(N) for short).

Considering S- and R-implications, their behaviour from this point of view is rather different. While any S-implication satisfies CPS(N) for any strong negation N (see [15]), this is not the case for R-implications in general.

First we formulate CPS(N) in some equivalent ways and state some basic connections between the implication and the negation on one hand, and between the t-norm and the negation on the other hand.

Theorem 4. Suppose that T is a left-continuous t-norm and N is a strong negation. Then the following three conditions are equivalent.

(a) I_T has contrapositive symmetry with respect to N;

(b) $I_T(x, y) = N(T(x, N(y)))$ for all $x, y \in [0, 1]$;

(c) $T(x, y) \leq z$ if and only if $T(x, N(z)) \leq N(y)$ for all $x, y, z \in [0, 1]$.

If I_T has CPS(N) then

(d) $N(x) = I_T(x, 0)$, $x \in [0, 1]$;

(e) $T(x, y) = 0$ if and only if $x \leq N(y)$, for $x, y \in [0, 1]$.

By the equivalence of statements (a) and (b), an R-implication can have CPS(N) if and only if it is at the same time an S-implication. By (e), only those R-implications can satisfy CPS(N) for which the underlying t-norm fulfils the law of contradiction with respect to N. In the case of continuous t-norms we have the following unicity result (see also [13]).

Theorem 5. Suppose that T is a continuous t-norm. Then I_T has contrapositive symmetry with respect to a strong negation N if and only if there exists an automorphism φ of the unit interval such that

$$T(x, y) = \varphi^{-1}(\max\{\varphi(x) + \varphi(y) - 1, 0\}), \tag{6}$$
$$N(x) = \varphi^{-1}(1 - \varphi(x)). \tag{7}$$

In this case I_T is given by

$$I_T(x, y) = \varphi^{-1}(\min\{1 - \varphi(x) + \varphi(y), 1\}). \tag{8}$$

5.1 Contrapositive Symmetrization of R-Implications

Suppose that T is a left-continuous t-norm and N is a strong negation. Define a new implication associated with I_T as follows:

$$x \rightarrow_T y := \max\{I_T(x, y), I_T(N(y), N(x))\}, \quad x, y \in [0, 1]. \tag{9}$$

If I_T has contrapositive symmetry then $x \rightarrow_T y = I_T(x, y) = I_T(N(y), N(x))$.

Define also a binary operation $*_T$ by

$$x *_T y := \min\{T(x, y), N[I_T(y, N(x))]\}, \quad x, y \in [0, 1]. \tag{10}$$

Obviously, $*_T = T$ if CPS(N) is satisfied by I_T. Even in the opposite case, this operation $*_T$ is a fuzzy conjunction in a broad sense and has several nice properties as we state in the next theorem.

Theorem 6. Suppose that T is a left-continuous t-norm and N is a strong negation such that $N(x) \geq I_T(x, 0)$ for all $x \in [0, 1]$ and operations $\rightarrow_T$ and $*_T$ are defined by (9) and (10), respectively. Then the following conditions are satisfied:

(a) $1 *_T y = y$;

(b) $x *_T 1 = x$;

(c) $*_T$ is nondecreasing in both arguments;

(d) $x \rightarrow_T y \geq z$ if and only if $x *_T z \leq y$.

It is interesting to know whether $*_T$ is also a t-norm for some T. A sufficient condition to assure this case is given in the next theorem.

Theorem 7. If $T(x, y) \leq N(I_T(y, N(x)))$ holds for $y > N(x)$, where T is a t-norm and N is a strong negation, then $*_T$ is also a t-norm.

As a consequence of this theorem, we have the following result.

Corollary 1. Let N be a strong negation and T be a t-norm such that $T(x, y) > 0$ when $y > N(x)$. Then the operation defined by

$$T_0(x, y) = \begin{cases} T(x, y) & \text{if } y > N(x) \\ 0 & \text{otherwise} \end{cases}$$

is a t-norm if and only if $T(x, y) \leq N[I_T(y, N(x))]$ for $y > N(x)$.

5.2 Nilpotent Minimum

If $T(x, y) = \min\{x, y\}$ then $x *_{\min} y$, denoted as $\min_0(x, y)$, is defined by

$$\min_0(x, y) = \begin{cases} \min(x, y) & \text{if } y > N(x) \\ 0 & \text{otherwise} \end{cases}$$

is a t-norm for any strong negation N since

$$\min(x, y) \leq n(I_{\min}(y, N(x))) = x$$

holds for $y > N(x)$. This t-norm is called *nilpotent minimum* with respect to the strong negation N (see [12, 6]).

Suppose that φ is an automorphism of the unit interval, and define a binary operation on $[0, 1]$ by

$$\min_\varphi(x, y) = \begin{cases} \min(x, y) & \text{if } \varphi(x) + \varphi(y) > 1 \\ 0 & \text{if } \varphi(x) + \varphi(y) \leq 1 \end{cases}.$$

Clearly, the following equivalent form of $\min_\varphi$ can be obtained by using the strong negation N^φ generated by φ:

$$\min_\varphi(x,y) = \begin{cases} \min(x,y) & \text{if } y > N^\varphi(x) \\ 0 & \text{otherwise.} \end{cases}$$

In the next theorem we list the most important properties of $\min_\varphi$ and $\max_\varphi$. These are easy to prove.

Theorem 8. Suppose that φ is an automorphism of the unit interval. The t-norm $\min_\varphi$ and the t-conorm $\max_\varphi$ have the following properties:

(a) The law of contradiction holds for $\min_\varphi$ as follows:

$$\min_\varphi(x, N^\varphi(x)) = 0 \quad \forall x \in [0,1].$$

(b) The law of excluded middle holds for $\max_\varphi$:

$$\max_\varphi(x, N^\varphi(x)) = 1 \quad \forall x \in [0,1].$$

(c) There exists a number α_0 depending on φ such that $0 < \alpha_0 < 1$ and $\min_\varphi$ is idempotent on the interval $(\alpha_0, 1]$:

$$\min_\varphi(x,x) = x \quad \forall x \in (\alpha_0, 1].$$

(d) With the previously obtained α_0, $\max_\varphi$ is idempotent on the interval $[0, \alpha_0)$:

$$\max_\varphi(x,x) = x \quad \forall x \in [0, \alpha_0).$$

(e) There exists a subset X_φ of the unit square such that $(x,y) \in X_\varphi$ if and only if $(y,x) \in X_\varphi$ and the law of absorption holds on X_φ as follows:

$$\max_\varphi(x, \min_\varphi(x,y)) = x \quad \forall (x,y) \in X_\varphi.$$

(f) There exists a subset Y_φ of the unit square such that $(x,y) \in Y_\varphi$ if and only if $(y,x) \in Y_\varphi$ and the law of absorption holds on Y_φ as follows:

$$\min_\varphi(x, \max_\varphi(x,y)) = x \quad \forall (x,y) \in Y_\varphi.$$

(g) If A, B are fuzzy subsets of the universe of discourse U and the α-cuts are denoted by A_α, B_α, respectively ($\alpha \in [0,1]$), then we have

$$A_\alpha \cap B_\alpha = [\min_\varphi(A,B)]_\alpha \quad \forall \alpha \in (\alpha_0, 1]$$

and

$$A_\alpha \cup B_\alpha = [\max_\varphi(A,B)]_\alpha \quad \forall\alpha \in [0,\alpha_0),$$

where α_0 is given in (c).

(h) $\min\varphi$ is a left-continuous t-norm and $\max_\varphi$ is a right-continuous t-conorm.

Proof. (a) and (b) are obviously true.

Concerning (c) and (d), define $\alpha_0 = \varphi^{-1}(1/2)$.

In case (e) define X_φ by

$$X_\varphi = \{(x,y) \in [0,1] \mid \varphi(x) + \varphi(y) \leq 1\}.$$

Similarly, in case (f) Y_φ can be defined as

$$Y_\varphi = \{(x,y) \in [0,1] \mid \varphi(x) + \varphi(y) \geq 1\}.$$

Case (g) follows from parts (c) and (d).

Finally, (h) is implied by the definition of $\min_\varphi$ and $\max_\varphi$, respectively. □

6 IMPLICATIONS DEFINED BY NILPOTENT MINIMUM AND NILPOTENT MAXIMUM

Consider the De Morgan triple $(\min_\varphi, \max_\varphi, N^\varphi)$ with an automorphism φ of the unit interval and define the corresponding S-implication:

$$\begin{aligned} I(x,y) &= \max_\varphi(N^\varphi(x), y) \\ &= \begin{cases} 1 & x \leq y \\ \max(N^\varphi(x), y) & x > y. \end{cases} \end{aligned}$$

Since the R-implication defined by $\min_\varphi$ coincides with this S-implication, $I_{\min_\varphi}$ always has contrapositive symmetry with respect to N^φ.

Now we list the most important and attractive properties of $I_{\min_\varphi}$. Their richness is due to the fact that R- and S-implications coincide and thus advantageous features of both classes are combined.

1. $I_{\min_\varphi}(x,.)$ is non-decreasing
2. $I_{\min_\varphi}(.,y)$ is non-increasing
3. $I_{\min_\varphi}(1,y) = y$
4. $I_{\min_\varphi}(0,y) = 1$
5. $I_{\min_\varphi}(x,1) = 1$
6. $I_{\min_\varphi}(x,y) = 1$ if and only if $x \leq y$
7. $I_{\min_\varphi}(x,y) = I_{\min_\varphi}(N^\varphi(y), N^\varphi(x))$
8. $I_{\min_\varphi}(x,0) = N^\varphi(x)$

9. $I_{\min_\varphi}(x, I_{\min_\varphi}(y, x)) = 1$
10. $I_{\min_\varphi}(x, .)$ is right-continuous
11. $I_{\min_\varphi}(x, x) = 1$
12. $I_{\min_\varphi}(x, I_{\min_\varphi}(y, z)) = I_{\min_\varphi}(y, I_{\min_\varphi}(x, z)) = I_{\min_\varphi}(\min_\varphi(x, y), z)$
13. $\min_\varphi(x, I_{\min_\varphi}(x, y)) \leq \min(x, y)$
14. $I_{\min_\varphi}(x, y) \geq \min(x, y)$.

Notice that $I_{\min_\varphi}$ can also be viewed as a QL-implication defined by

$$\begin{aligned} S(x, y) &= \max_\varphi(x, y), \\ N(x) &= N^\varphi(x) \\ T(x, y) &= \min(x, y) \end{aligned}$$

as one can check easily by simple calculus.

Therefore, this QL-implication (which is, in fact, an S-implication and an R-implication at the same time) also has contrapositive symmetry with respect to N^φ. Concerning this case, the following unicity result was proved in [6].

Theorem 9. Consider a QL-implication defined by $\max_\varphi(N^\varphi(x), T(x, y))$, with a t-norm T. This implication has contrapositive symmetry with respect to N^φ if and only if $T = \min$.

7 FUZZY INFERENCE SYSTEMS AND FUZZY IMPLICATIONS

Fuzzy inference systems generate inference results based on fuzzy if-then rules. Fuzzy implications are mostly used as a way of interpretation of the if-then rules with fuzzy antecedent and/or fuzzy consequent.

Fuzzy if-then rules may be interpreted in two ways: as a conjunction of the antecedent and the consequent (Mamdani combination) or in the spirit of the classical logical implication, i.e. as a fuzzy implication.

Approximate reasoning is usually executed in a fuzzy inference system which performs a mapping from an input fuzzy set to a fuzzy set via a fuzzy rule base. Two methods of approximate reasoning are mostly used: composition based inference (first aggregate then inference (FATI)) and individual-rule based inference (first inference then aggregate (FITA)).

In composition-based inference, a finite number of rules is aggregated via appropriate aggregation operations (like intersections or means).

Taking into account an arbitrary input fuzzy set and using the generalized modus ponens we obtain the output of fuzzy inference (FATI) in a closed form. In individual-rule-based inference (FITA) each rule in the fuzzy rule base determines an output fuzzy set and after that an aggregation via intersection or average operation is performed.

An output fuzzy set obtained from inference system based on fuzzy implication interpretation of if-then rules is different from the resulting fuzzy set obtained from inference system based on conjunctive interpretation of fuzzy if-then rules.

8 CONCLUSION

The nilpotent minimum and the corresponding implication combine advantageous properties of Łukasiewicz-like t-norms (e.g. the law of contradiction holds, the corresponding R- and S-implications coincide) and those of the minimum itself (e.g. easy usage of α-cuts in practice). By these propitious characteristics, we hope that the results of the present paper will urge practical users of fuzzy logic to apply new operations (especially nilpotent minimum and its associated implication) for modeling problems in important fields such as fuzzy control, engineering and hardware implementations.

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