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SYNTHESIS OF LIVENESS-ENFORCING PETRI NET SUPERVISORS BASED ON A THINK-GLOBALLY-ACT-LOCALLY APPROACH AND A STRUCTURALLY MINIMAL METHOD FOR FLEXIBLE MANUFACTURING SYSTEMS

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Abstract. This paper proposes a deadlock prevention policy for flexible manufacturing systems (FMSs) based on a think-globally-act-locally approach and a structurally minimal method. First, by using the think-globally-act-locally approach, a global idle place is temporarily added to a Petri net model with deadlocks. Then, at each iteration, an integer linear programming problem is formulated to design a minimal number of maximally permissive control places. Therefore, a supervisor with a low structural complexity is obtained since the number of control places is

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greatly compressed. Finally, by adding the designed supervisor, the resulting net model is optimally or near-optimally controlled. Three examples from the literature are used to illustrate the proposed method.

Keywords: Flexible manufacturing system, deadlock prevention, think-globally-act-locally approach, structurally minimal method, maximal permissiveness

Mathematics Subject Classification 2010: 93-C65

1 INTRODUCTION

A flexible manufacturing system (FMS) performs different kinds of tasks with multiple processes that compete for limited resources, such as robots, machines, automated guided vehicles, buffers, fixtures, etc. In an FMS, deadlock may be caused by the competition of system resources between different processes [1]. Generally, a deadlock makes a system blocked and inefficient, even leads to destructive results, which is highly undesirable. Therefore, a number of approaches, such as deadlock detection and recovery [2, 3], deadlock avoidance [4, 5, 6], and deadlock prevention [7, 8, 9, 10, 11, 12], have been proposed to resolve the deadlock issue. In this paper, we focus on discussing deadlock prevention policies.

Petri nets, graph theory, and automata are generally considered to deal with deadlocks in FMSs [13, 14, 15, 16]. Compared with graph theory and automata, Petri nets have some irreplaceable advantages in dealing with deadlocks [17, 18, 19, 20, 21, 22]. In an FMS modeled by Petri nets, only some simple and necessary constraints are required to prevent deadlocks, and a supervisor can be obtained offline in a static way [23, 24, 25, 26, 27]. Some major indicators are considered to design liveness-enforcing Petri net supervisors, including behavioral permissiveness, structural complexity, and computational complexity. Permissive behavior represents the legal markings of a Petri net. Hence, a supervisor is said to be maximally permissive if all legal markings are reachable. The structural complexity is always evaluated by the number of control places in a supervisor. Computational complexity indicates the efficiency of the proposed algorithm to design a supervisor.

Generally, the techniques used for synthesizing deadlock prevention policies can be divided into two types: structural analysis [28, 29, 30, 31] and reachability graph analysis [7, 10, 32, 33, 34]. Structural analysis is an effective way to deal with deadlocks for some special Petri net structures (for example siphons and resource circuits) [35]. However, it usually leads to that the resulting net model is not optimally controlled [36, 37], since the computed supervisor is so conservative that many legal behaviors of the system are prevented. Compared with structural analysis techniques, the reachability graph analysis approaches can lead to optimal or near-optimal supervisors for generalized Petri net models. Nevertheless, these methods need to enumerate all reachable markings of a system [10, 38, 39, 40].

B	number of tokens in the GP
DZ	deadlock-zone
FBM	first-met bad marking
FMS	flexible manufacturing system
GP	global idle place
$G(N, M_0)$	reachability graph of net (N, M_0)
I	place invariant
ILPP	integer linear programming problem
LZ	live-zone
M	marking
$\mathcal{M}_{\mathrm{L}}$	set of legal markings
$\mathcal{M}_{\mathrm{FBM}}$	set of FBMs
$\mathcal{M}_{\mathrm{L}}^{\star}$	minimal covering set of legal markings
$\mathcal{M}_{\mathrm{FBM}}^{\star}$	minimal covered set of FBMs
MCPP	minimal number of control places problem
N	Petri net with $N = (P, T, F, W)$
N_B	Petri net with the GP
$\mathbb{N}$	set of non-negative integers
$[N]$	incidence matrix
(N, M_0)	Petri net model
p	place in a Petri net
p_s	control place (monitor)
P	set of places in a Petri net
PI	place invariant
Q	big enough integer constant
$R(N, M_0)$	set of reachable markings of net (N, M_0)
t	transition in a Petri net
T	set of transitions in a Petri net
TGAL	think-globally-act-locally method
TGALW	think-globally-act-locally method with weighted arcs
$^{\bullet}x$	preset of a node $x \in P \cup T$
$x^{\bullet}$	postset of a node $x \in P \cup T$

Table 1. Nomenclature

In this paper, we mainly discuss methods related to the reachability graph analysis. Given a control specification, all markings of a system can be divided into two categories: legal and illegal ones. For the specification of deadlock prevention, a marking is called as a legal marking if itself or one of its succeeding markings can evolve back to the initial marking, otherwise, it is an illegal marking. A control policy is optimal if it ensures that all illegal markings are prohibited while no legal marking is prevented.

In [41, 42], a reachability graph is partitioned into a live-zone (LZ) and a deadlock-zone (DZ), and these two zones contain all legal and illegal markings, respectively. Then, a first-met bad marking (FBM) is defined as an illegal one that represents the very first entry from the LZ to the DZ. FBMs are a special kind of illegal markings related to deadlocks, since the system cannot enter into the DZ if they are all prevented. However, some legal markings may be prohibited when a set of control places (monitors) is computed to prevent all FBMs. That is to say, the obtained supervisor cannot be ensured to be behaviorally optimal and also suffers from the structural complexity problem due to too many control places obtained.

To solve the above problem, Chen et al. propose a vector covering approach by analyzing the relationship between different markings [38]. They first put forward the concept of two minimal sets: a minimal covering set of legal markings and a minimal covered set of FBMs, which can be used to design control places without considering all legal markings and all FBMs. However, there are too many control places in a supervisor since a monitor is required to be designed for each FBM in the minimal covered set of FBMs.

In [10], Chen and Li further develop an approach to design structurally minimal controller, which selects as few control places as possible. By using this method, no redundant monitor survives [7, 10]. Also, it ensures that the obtained supervisor is behaviorally optimal. Nevertheless, for a complex net model, it is impossible to compute a maximally permissive supervisor in a reasonable time by using this method due to the complexity of solving an integer linear program with too many constraints and variables.

In [44], Uzam et al. present a think-globally-act-locally method (TGAL) to prevent deadlocks in an iterative way, which can alleviate the state explosion problem. They propose a global idle place (GP), which is temporarily added to the original Petri net model but the GP does not change any of its basic properties. At each iteration, by increasing one token in the GP, the reachability graph of the related net is generated and a set of control places is computed to prevent deadlocks. Finally, all monitors designed in the iteration processes are added to the original net model such that the resulting net model is live. However, this method cannot ensure the maximal permissiveness of computed supervisors for generalized Petri net models since some legal markings may be lost in the iteration processes.

In order to improve TGAL, Uzam et al. further develop a think-globally-act-locally approach with weighted arcs (TGALW) [45]. Compared with their previous work, it can obtain control places with weighted arcs at each iteration by transforming the original Petri net into a strictly conservative form. This method ensures that the resulting controlled system has more reachable markings than the previous TGAL. However, it also faces the problem that the designed supervisor is not optimal for generalized net models.

Similarly, the method previously proposed by the authors of this work [46] improves the behavioral permissiveness of TGAL. At each iteration, a vector covering approach is applied to design a set of control places with maximally permissiveness. By using this method, the resulting Petri net models have more reachable behaviors than TGAL and TGALW, and in most of cases it is optimal. Nevertheless, it also has some drawbacks, such as the obtained supervisor has too many control places and a redundancy test is necessary to select the necessary monitors at each iteration.

In this paper, we further develop a method for supervisory control based on TGAL and the structural minimization of a controller. At each iteration, the structurally minimal method is used to formulate an integer linear programming problem (ILPP). By solving this ILPP, a set of optimal control places is obtained with the number of control places being minimized. Therefore, the number of designed monitors is greatly compressed and the redundancy test is not necessary at each iteration. Finally, the resulting net model is live by adding a small number of control places. Compared with TGAL, TGALW, and our previous work [46], this approach constructs an optimal or near-optimal supervisor with fewer control places.

We organize the rest of this paper as follows. Section 2 outlines some basic concepts used in this paper, such as Petri nets [17], control place synthesis by place invariant (PI) [43], and the structurally minimal approach [10]. A deadlock prevention policy is proposed to design supervisors with simple structures in Section 3. Section 4 presents some experiment results by using the proposed method. Finally, conclusions are given in Section 5.

2 PRELIMINARIES

2.1 Petri Nets

A Petri net [17] is a four-tuple $N = (P, T, F, W)$, where P and T are finite and non-empty sets of places and transitions, respectively. The flow relation of a net is represented by arcs with arrows from places to transitions or from transitions to places, denoted as $F \subseteq (P \times T) \cup (T \times P)$. $W : (P \times T) \cup (T \times P) \to \mathbb{N}$ is a mapping that assigns a weight to an arc: $W(x, y) > 0$ if $(x, y) \in F$, and $W(x, y) = 0$, otherwise, where $x, y \in P \cup T$ and $\mathbb{N}$ is the set of non-negative integers. The preset and postset of a node $x \in P \cup T$ are ${}^\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$ and $x^\bullet = \{y \in P \cup T \mid (x, y) \in F\}$, respectively. A marking represents a mapping $M : P \to \mathbb{N}$. The number of tokens in place p at marking M is denoted as $M(p)$. Generally, vector M can be written as $\sum_{p \in P} M(p)p$. The pair (N, M_0) is called a marked Petri net or a net system. Incidence matrix $[N]$ of net N is a $|P| \times |T|$ integer matrix with $[N](p, t) = W(t, p) - W(p, t)$.

A transition $t \in T$ is enabled at marking M if for all $p \in {}^\bullet t$, $M(p) \geq W(p, t)$. This fact is denoted as $M[t\rangle$. Once a transition t fires, it yields a marking M', denoted as $M[t\rangle M'$, where $M'(p) = M(p) - W(p, t) + W(t, p)$, for all $p \in P$. $M_0[\rangle$ is called the set of reachable markings of a Petri net N from the initial marking M_0, often denoted by $R(N, M_0)$. A reachability graph is a graphical representation of $R(N, M_0)$. The reachability graph of a net (N, M_0), denoted as $G(N, M_0)$, is a directed graph whose nodes are markings in $R(N, M_0)$ and arcs are labeled by the transitions of N.

Let (N, M_0) be a net system with $N = (P, T, F, W)$. A transition $t \in T$ is live at M_0 if there exists $M' \in R(N, M)$ such that $M'[t\rangle$, for all $M \in R(N, M_0)$. (N, M_0) is live if for all $t \in T$, t is live at M_0. It is dead at M_0 if there does not exist $t \in T$ such that $M_0[t\rangle$.

A P-vector is a column vector $I : P \to \mathbb{Z}$ indexed by P, where $\mathbb{Z}$ is the set of integers. P-vector I is called a place invariant (PI for short) if $I \neq \mathbf{0}$ and $I^T[N] = \mathbf{0}^T$. Let I be a PI of (N, M_0) and M be a reachable marking from M_0. Then, $I^T M = I^T M_0$.

2.2 Analysis of Reachability Graph

For the optimal control purpose, a supervisor should ensure that all legal markings are reachable. In a Petri net model (N, M_0), set $\mathcal{M}_\mathrm{L}$ consists of all legal markings, denoted as:

$$\mathcal{M}_\mathrm{L} = \{M | M \in R(N, M_0) \wedge M_0 \in R(N, M)\}. \tag{1}$$

A reachability graph can be partitioned into two zones: a deadlock-zone (DZ) and a live-zone (LZ) [41, 42]. A first-met bad marking (FBM) is a special illegal marking that can be firstly met from the LZ to the DZ by firing one transition. The set of FBMs is defined as:

$$\mathcal{M}_\mathrm{FBM} = \{M \in \mathrm{DZ} | \exists M' \in \mathrm{LZ}, \exists t \in T, M'[t\rangle M\}. \tag{2}$$

2.3 Control Place Synthesis Method by Place Invariant

In [43], Yamalidou et al. propose an approach to compute a control place (or monitor) by a PI, including its initial marking and connected arcs. Let $[N_0]$ be the incidence matrix of a Petri net to be controlled with n places and m transitions. Given a control requirement, the following constraint requires to be satisfied:

$$\sum_{i=1}^{n} l_i \cdot M(p_i) \leq \beta \tag{3}$$

where l_i and β are non-negative integers, and $M(p_i)$ represents the marking of place p_i. By introducing a non-negative slack variable $M(p_s)$, Equation (3) is transformed as follows:

$$\sum_{i=1}^{n} l_i \cdot M(p_i) + M(p_s) = \beta \tag{4}$$

where p_s is a monitor. In particular, at the initial marking, Equation (4) can be written as:

$$M_0(p_s) = \beta - \sum_{i=1}^{n} l_i \cdot M_0(p_i) \tag{5}$$

where $M_0(p_s)$ is the initial marking of the control place p_s.

2.4 Optimal Control Place Synthesis

A manufacturing-oriented Petri net (M-net for short) is proposed in [49], which is a generalization of the existing net classes that model FMS. In an M-net, there are three categories of places: idle, operation (activity), and resource places, whose sets are denoted as P^0, P_A, and P_R, respectively [23, 48]. An idle place means a raw part before entering a production sequence. The tokens in an idle place represent the number of concurrent operations that can happen in the production sequences. An operation place indicates an operation to be processed for a part in a production sequence and initially it has no token. A resource place models a type of available resources, such as robots and machines. At the initial state, tokens in a resource place are equal to the number of available resource units.

By considering the tokens in operation places only, we construct a PI to prohibit an FBM [41]. In this paper, $\mathbb{N}_A$ is used to denote $\{i|p_i \in P_A\}$. An FBM $M_f \in \mathcal{M}_{\mathrm{FBM}}$ should be prevented by enforcing the following constraint:

$$\sum_{i\in\mathbb{N}_A} l_i \cdot \mu_i \leq \beta \tag{6}$$

where

$$\beta = \sum_{i\in\mathbb{N}_A} l_i \cdot M_f(p_i) - 1. \tag{7}$$

Equation (6) is the forbidden condition. For a legal marking $M' \in \mathcal{M}_{\mathrm{L}}$, the reachability condition is given as follows:

$$\sum_{i\in\mathbb{N}_A} l_i \cdot M'(p_i) \leq \beta, \quad \forall M' \in \mathcal{M}_{\mathrm{L}}. \tag{8}$$

By substituting the β in Equation (7) into Equation (8), the following equation is obtained:

$$\sum_{i\in\mathbb{N}_A} l_i \cdot (M'(p_i) - M_f(p_i)) \leq -1, \quad \forall M' \in \mathcal{M}_{\mathrm{L}}. \tag{9}$$

By solving Equation (9), we can obtain a set of feasible solutions for coefficients l_i's. Then, an optimal PI is computed, which can ensure that an FBM is prevented while all legal markings are reachable.

In order to further reduce the number of markings in $\mathcal{M}_{\mathrm{L}}$ and $\mathcal{M}_{\mathrm{FBM}}$, a vector covering technique [38] is proposed, and more details of this method are given as follows:

Definition 1 ([38])**.** Let M and M' be two markings in $R(N, M_0)$. M A-covers M' (or M' is A-covered by M) if $M(p) \geq M'(p)$, for all $p \in P_A$, which is denoted as $M \geq_A M'$(or $M' \leq_A M$).

Definition 2 ([38])**.** Let $\mathcal{M}_{\mathrm{L}}^{\star}$ be a subset of legal markings. $\mathcal{M}_{\mathrm{L}}^{\star}$ is called a minimal covering set of legal markings if the following two conditions are satisfied:

1. $(\forall M \in \mathcal{M}_{\mathrm{L}})(\exists M' \in \mathcal{M}_{\mathrm{L}}^{\star})M' \geq_A M$; and
2. $(\forall M \in \mathcal{M}_{\mathrm{L}}^{\star})(\nexists M' \in \mathcal{M}_{\mathrm{L}}^{\star})M' \geq_A M \ \& \ M \neq M'$.

Definition 3 ([38])**.** Let $\mathcal{M}^{\star}_{\rm FBM}$ be a subset of $\mathcal{M}_{\rm FBM}$. $\mathcal{M}^{\star}_{\rm FBM}$ is called a minimal covered set of FBMs if the following two conditions are satisfied:

1. $(\forall M_f \in \mathcal{M}_{\rm FBM})(\exists M'_f \in \mathcal{M}^{\star}_{\rm FBM}) M_f \geq_A M'_f$; and
2. $(\forall M_f \in \mathcal{M}^{\star}_{\rm FBM})(\nexists M'_f \in \mathcal{M}^{\star}_{\rm FBM}) M_f \geq_A M'_f$ & $M_f \neq M'_f$.

Corollary 1 ([38])**.** If all markings in $\mathcal{M}^{\star}_{\rm FBM}$ are forbidden by PIs, then all FBMs are forbidden.

Corollary 2 ([38])**.** If any marking in $\mathcal{M}^{\star}_{\rm L}$ is not prevented by PIs, then all legal markings are reachable.

According to Corollaries 1 and 2, an optimal supervisor is computed by using markings in sets $\mathcal{M}^{\star}_{\rm L}$ and $\mathcal{M}^{\star}_{\rm FBM}$. Finally, Equation (9) can be simplified as follows:

$$\sum_{i \in \mathbb{N}_A} l_i \cdot (M'(p_i) - M_f(p_i)) \leq -1, \quad \forall M' \in \mathcal{M}^{\star}_{\rm L}. \tag{10}$$

2.5 Minimal Supervisory Structure Synthesis

Similarly, we use $\mathbb{N}^{\star}_{\rm FBM}$ to denote $\{i | M_i \in \mathcal{M}^{\star}_{\rm FBM}\}$ in this paper. Given two FBMs M_j and M_k ($j, k \in \mathbb{N}^{\star}_{\rm FBM}$ and $j \neq k$), a PI I_j is designed to prevent M_j. Then, M_k is forbidden if the following constraint is satisfied:

$$\sum_{i \in \mathbb{N}_A} l_{j,i} \cdot (M_k(p_i) - M_j(p_i)) \geq 0 \tag{11}$$

where $l_{j,i}$'s represent the coefficients of PI I_j.

A set of binary variables $f_{j,k}$'s ($j \neq k$) is introduced to determine whether M_k can be prevented by PI I_j or not. That is, M_k is forbidden by PI I_j if $f_{j,k} = 1$, otherwise, $f_{j,k} = 0$. Equation (11) is transformed as follows:

$$\sum_{i \in \mathbb{N}_A} l_{j,i} \cdot (M_k(p_i) - M_j(p_i)) \geq -\mathrm{Q} \cdot (1 - f_{j,k}), \quad \forall M_k \in \mathcal{M}^{\star}_{\rm FBM} \text{ and } j \neq k \tag{12}$$

where Q is an integer constant that must be big enough and $f_{j,k} \in \{0, 1\}$.

Then, by introducing a set of binary variables h_j's, we have the following constraint for PI I_j:

$$f_{j,k} \leq h_j, \quad \forall j,\ k \in \mathbb{N}^{\star}_{\rm FBM} \text{ and } j \neq k \tag{13}$$

where $h_j \in \{0, 1\}$. $h_j = 1$ means that PI I_j is selected to prevent FBM M_j, otherwise, PI I_j is redundant and $h_j = 0$.

For FBM M_j, we should ensure that at least one PI is selected to forbid it and the following constraint is obtained:

$$h_j + \sum_{k \in \mathbb{N}^{\star}_{\rm FBM},\ j \neq k} f_{k,j} \geq 1. \tag{14}$$

By grouping Equations (10), (11), (12), (13), and (14), we can formulate the following ILPP, namely the minimal number of control places problem (MCPP).

$$\min \sum_{j \in \mathbb{N}^{\star}_{\mathrm{FBM}}} h_j,$$

subject to

$$\sum_{i \in \mathbb{N}_A} l_{j,i} \cdot (M'(p_i) - M_j(p_i)) \leq -1,$$

$$\forall M' \in \mathcal{M}^{\star}_{\mathrm{L}} \text{ and } \forall M_j \in \mathcal{M}^{\star}_{\mathrm{FBM}}, \tag{15}$$

$$\sum_{i \in \mathbb{N}_A} l_{j,i} \cdot (M_k(p_i) - M_j(p_i)) \geq -\mathrm{Q} \cdot (1 - f_{j,k}),$$

$$\forall M_j, M_k \in \mathcal{M}^{\star}_{\mathrm{FBM}} \text{ and } k \neq j, \tag{16}$$

$$f_{j,k} \leq h_j, \forall j, k \in \mathbb{N}^{\star}_{\mathrm{FBM}} \text{ and } j \neq k, \tag{17}$$

$$h_j + \sum_{k \in \mathbb{N}^{\star}_{\mathrm{FBM}},\ j \neq k} f_{k,j} \geq 1, \tag{18}$$

$$l_{j,i} = \{0, 1, 2, \dots\}, \forall i \in \mathbb{N}_A \text{ and } \forall j \in \mathbb{N}^{\star}_{\mathrm{FBM}},$$

$$f_{j,k} \in \{0, 1\}, \forall k, j \in \mathbb{N}^{\star}_{\mathrm{FBM}} \text{ and } k \neq j,$$

$$h_j \in \{0, 1\}, \forall j \in \mathbb{N}^{\star}_{\mathrm{FBM}}.$$

The above MCPP can obtain a behaviorally optimal and structurally minimal supervisor since all legal markings are reachable and the number of control places is minimized.

3 DEADLOCK PREVENTION POLICY

This section proposes a method based on TGAL approach and the structurally minimal technique to prevent deadlocks. At each iteration, the structurally minimal method is used to design a set of maximally permissive control places and compress the number of them. The main advantage is that it can design an optimal or near-optimal supervisor with a small number of control places. Finally, the proposed approach is shown as follows, denoted as Algorithm 1.

In Algorithm 1, we first design a GP to be added to a Petri net model with deadlocks. The related net with the GP is denoted as N_B, where B is the number of tokens in the GP. Initially, $B = 1$, and the related net N_1 is selected to compute its reachability graph. If N_1 has deadlocks, the structurally minimal approach is used to design optimal control places and minimize the number of them. By adding the designed control places, the net N_1 is live. Then, we increase one token in the GP

Algorithm 1 A deadlock prevention policy based on TGAL method and the structurally minimal approach

Require: Petri net model (N, M_0) for an FMS with $N = (P^0 \cup P_A \cup P_R, T, F, W)$.
Ensure: A controlled Petri net model (N^α, M_0^α).

Compute input and output transitions of idle places, and obtain sets $T_I = {}^\bullet P^0$ and $T_O = P^{0\bullet}$.
Design a global idle place (GP) with $\text{GP}^\bullet = T_O$, ${}^\bullet\text{GP} = T_I$, and $M_0(\text{GP}) = B$, then the Petri net model with the GP is denoted as N_B. /* B represents the number of tokens in the GP. */
Initialize $B = 1$ and $V_M := \emptyset$. /* V_M is a set of control places designed for (N, M_0). */
while $\{B \leq K\}$ **do** /* K means the sum of initial tokens of all idle places. */
  Compute the reachability graph of N_B.
  if $\{N_B$ is not live$\}$ **do**
    Compute FBMs and legal markings of N_B, i.e., sets $\mathcal{M}_{\text{FBM}}$ and $\mathcal{M}_{\text{L}}$, respectively.
    Calculate the minimal covering sets of FBMs and legal markings, namely $\mathcal{M}^\star_{\text{FBM}}$ and $\mathcal{M}^\star_{\text{L}}$, respectively.
    Formulate an ILPP, i.e., the MCPP developed in Section 2.5.
    Solve the MCPP to find an optimal solution of h_j, and obtain the corresponding values of $l_{j,i}$ and β_j $(i \in \mathbb{N}_A,\ j \in \mathbb{N}^\star_{\text{FBM}})$.
    foreach $\{h_j = 1\}$ **do**
      Design a control place p_{s_j} by using the solution of $l_{j,i}$ as the PI I_j's coefficients.
      $V_M := V_M \cup \{p_{s_j}\}$.
    endforeach
  endif
  B++;
endwhile
Add all monitors in V_M to the net model (N, M_0), and obtain a controlled net model (N^α, M_0^α).
Output the controlled Petri net model (N^α, M_0^α).
End.

$(B = 2)$, and repeat the above steps to make N_2 live. This process is terminated if no new reachable marking is generated by increasing the tokens in the GP. Finally, an optimal or near-optimal controlled net model is obtained, by adding a small number of control places.

Now we discuss the complexity of Algorithm 1. The proposed method needs to generate the reachability graph, in which the number of nodes increases exponentially with respect to the size of the net model. Then, an ILPP, namley the MCPP, is solved at each iteration, which is an NP-hard problem. Therefore, the complexity of this method is in theory exponential although only a partial reachability graph is computed. Compared with the technique in [10], this method reduces in general the computational overhead since only a part of reachability graph is generated at each iteration.

Theorem 1. Let N_B be a related net that is generated in the iteration processes of Algorithm 1, and $h^\star$ be an optimal solution of the MCPP that is formulated for N_B. Then, the net N_B is optimally controlled by a set of maximally permissive control places, and the number of them is minimized, if such an optimal solution $h^\star$ exists.

Proof. Suppose that $h^\star$ is an optimal solution of the MCPP. According to this solution, a set of PIs is obtained, then a set of associated control places is designed. We just need to show the obtained control places are maximally permissive and the number of them is minimized. In the MCPP, each PI should satisfy Equation (15), i.e., it can prevent an FBM in $\mathcal{M}^\star_{\mathrm{FBM}}$ while all legal markings are reachable. There does not exist a legal making prevented by PIs. By combining Equations (16) and (18), it ensures that each FBM in $\mathcal{M}^\star_{\mathrm{FBM}}$ is prevented by at least one PI. All FBMs in $\mathcal{M}^\star_{\mathrm{FBM}}$ are prevented by PIs, namely all FBMs are forbidden. Therefore, the obtained control places are maximally permissive. Equation (17) means that FBMs are prevented only by the selected PIs. Meanwhile, the objective function is applied to find the minimal number of selected PIs, i.e., the optimal solution $h^\star$ is equal to the minimal number of PIs required. Thus, the conclusion holds. □

A Petri net model of an FMS from [32] shown in Figure 1 is used as an example to illustrate the proposed method. It has 11 places and eight transitions, where $P^0 = \{p_1, p_8\}$, $P_A = \{p_2 - p_7\}$, and $P_R = \{p_9, p_{10}, p_{11}\}$. There are 18 reachable markings, including 15 legal markings and three FBMs. A GP is designed with ${}^\bullet\mathrm{GP} = T_I = {}^\bullet P^0 = \{t_4, t_8\}$ and $\mathrm{GP}^\bullet = T_O = P^{0\bullet} = \{t_1, t_5\}$. By introducing the GP into the net model, the related net N_B is obtained.

At the first iteration, the GP has only one token (namely $B = 1$). By generating the reachability graph of N_1, we find that it has seven legal markings but no deadlocks. Then, one more token is added into the GP ($B = 2$) and the related net N_2 is obtained. The net N_2 has 13 legal markings and two FBMs. Then, by applying the vector covering approach, we have $\mathcal{M}^\star_{\mathrm{L}} = \{p_3 + p_4, p_7 + p_8, p_2 + p_4, p_6 + p_8, p_2 + p_3, p_6 + p_7\}$ and $\mathcal{M}^\star_{\mathrm{FBM}} = \{p_2 + p_7, p_2 + p_6\}$.

For FBM $M_{f_1} = p_2 + p_7$, a PI I_1 can be designed to forbid it. According to Equation (15), I_1 has to satisfy the following six constraints: $l_{1,2} \cdot (0-1) + l_{1,3} \cdot (1-0) + l_{1,4} \cdot (1-0) + l_{1,7} \cdot (0-1) \leq -1$, $l_{1,2} \cdot (0-1) + l_{1,7} \cdot (1-1) + l_{1,8} \cdot (0-1) \leq -1$, $l_{1,2} \cdot (1-1) + l_{1,4} \cdot (1-0) + l_{1,7} \cdot (0-1) \leq -1$, $l_{1,2} \cdot (0-1) + l_{1,6} \cdot (1-0) + l_{1,7} \cdot (0-1) + l_{1,8} \cdot (1-0) \leq -1$, $l_{1,2} \cdot (1-1) + l_{1,3} \cdot (1-0) + l_{1,7} \cdot (0-1) \leq -1$, and $l_{1,2} \cdot (0-1) + l_{1,6} \cdot (1-0) + l_{1,7} \cdot (1-1) \leq -1$. By simplifying the above constraints, we have:

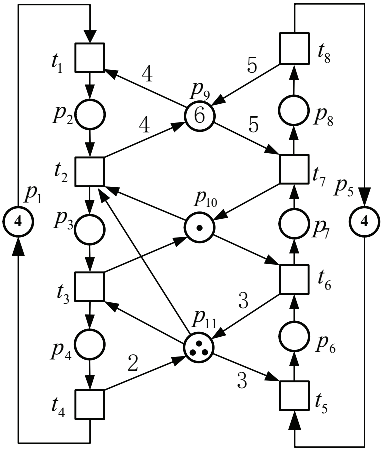

Figure 1. A Petri net model from [32]

$$-l_{1,2} + l_{1,3} + l_{1,4} - l_{1,7} \leq -1,$$
$$-l_{1,2} - l_{1,8} \leq -1,$$
$$l_{1,4} - l_{1,7} \leq -1,$$
$$-l_{1,2} + l_{1,6} - l_{1,7} + l_{1,8} \leq -1,$$
$$l_{1,3} - l_{1,7} \leq -1,$$
$$-l_{1,2} + l_{1,6} \leq -1.$$

Then, a variable $f_{1,2}$ ($f_{1,2} \in \{0, 1\}$) is introduced to indicate whether I_1 can prevent $M_{f_2} = p_2 + p_6$ or not. By Equation (16), we have $l_{1,2} \cdot (1-1) + l_{1,6} \cdot (1 - 0) + l_{1,7} \cdot (0-1) \geq -\mathrm{Q} \cdot (1 - f_{1,2})$, where Q is a big enough integer constant. The following constraint is obtained:

$$l_{1,6} - l_{1,7} \geq -\mathrm{Q} \cdot (1 - f_{1,2}).$$

Similarly, for $M_{f_2} = p_2 + p_6$, we have the following constraints:

$$-l_{2,2} + l_{2,3} + l_{2,4} - l_{2,6} \leq -1,$$
$$-l_{2,2} - l_{2,6} + l_{2,7} + l_{2,8} \leq -1,$$
$$l_{2,4} - l_{2,6} \leq -1,$$
$$-l_{2,2} + l_{2,8} \leq -1,$$

$$l_{2,3} - l_{2,6} \leq -1,$$

$$-l_{2,2} + l_{2,7} \leq -1,$$

and

$$l_{2,7} - l_{2,6} \geq -\mathrm{Q} \cdot (1 - f_{2,1}).$$

A set of variables h_j's ($h_j \in \{0, 1\}$ and $j \in \{1, 2\}$) is introduced to show whether PI I_j is selected to design a control place or not, i.e., $h_j = 1$ represents that PI I_j is selected, otherwise, I_j is redundant. According to Equation (17), the constraints between $f_{j,k}$ and h_j ($k, j \in \{1, 2\}$ and $k \neq j$) are obtained:

$$f_{1,2} \leq h_1,$$

$$f_{2,1} \leq h_2.$$

On the other hand, since at least one PI is required for each FBM, we have the following constraints:

$$h_1 + f_{2,1} \geq 1,$$

$$h_2 + f_{1,2} \geq 1.$$

After given the above constraints, the following objective function is designed to minimize the number of control places obtained:

$$\min = h_1 + h_2.$$

Finally, by combining the above constraints and the objective function, an MCPP is formulated. By solving this MCPP, an optimal solution is obtained, i.e., $h_1 = 1$ and $h_2 = 0$. It means that only I_1 is selected to design a control place. By $f_{1,2} = 1$, I_1 can also prevent M_{f_2}. Thus, the coefficients of I_1, $l_{1,2} = 1$, $l_{1,6} = 1$, and $l_{1,7} = 1$, are used to design a control place p_{s_1}. We have $\mu_2 + \mu_6 + \mu_7 + \mu_{p_{s_1}} = 1$, namely $M_0(p_{s_1}) = 1$, ${}^\bullet p_{s_1} = \{t_2, t_7\}$, and $p_{s_1}^\bullet = \{t_1, t_5\}$. By adding only one control place p_{s_1}, the net N_2 is live.

Next, the monitor p_{s_1} is applied to the related net and the number of tokens in the GP is increased by one ($B = 3$). We find that the net N_3 is live with 15 legal markings. If the tokens in the GP are increased, no new reachable marking is generated. Then, the iteration process is terminated and the GP can be removed. By adding only one control place p_{s_1} to the original net model, the net model is optimally controlled, as shown in Figure 2. Table 2 shows the iteration processes for this example. In this table, the first column represents the tokens in the GP, and the second column gives the control places added to N_B. The third and fourth columns indicate whether N_B has deadlocks or not and the number of reachable markings of N_B, respectively. The fifth column shows the markings in the DZ and other markings in the LZ are given in the sixth column. The last column presents the control places designed.

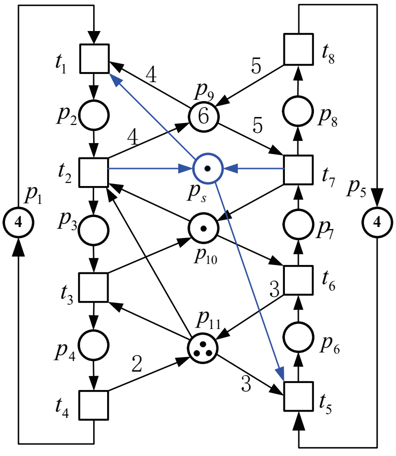

Figure 2. An optimally controlled system of the net in Figure 1

B	Include p_{s_i}	N_B Is Live?	Reachable Markings	DZ	LZ	p_{s_i}
1		Yes	7		7	
2		No	15	2	13	p_{s_1}
3	p_{s_1}	Yes	15		15	

Table 2. The iteration steps for the net shown in Figure 1 by Algorithm 1

4 EXPERIMENTAL RESULTS

In this section, we present some examples to show the application of the proposed method. C++ programs are designed to compute sets $\mathcal{M}^\star_{\text{FBM}}$ and $\mathcal{M}^\star_{\text{L}}$, and generate MCPPs. Then, we can use Lingo to solve MCPPs.

Figure 3 shows a Petri net model from [23]. In this net model, we have $P^0 = \{p_1, p_5, p_{14}\}$, $P_A = \{p_2 - p_4, p_6 - p_{13}, p_{15} - p_{19}\}$, and $P_R = \{p_{20} - p_{26}\}$. It has 26 750 reachable markings with 21 581 legal markings and 5 169 illegal markings. By applying the proposed approach to this net, Table 3 shows the iteration steps.

Table 4 compares the performance of different deadlock control policies. In this table, we can find that only five control places are required by using the method developed in [10]. However, it costs more than 40 hours to obtain an optimal solution. Compared with it, the proposed method only takes 35 seconds to obtain a sub-optimal solution in terms of the number of monitors. Meanwhile, the number of monitors designed by the proposed method is less than the one obtained by the work in [46]. It illustrates that the proposed method performs better in terms of computing time and the number of designed control places.

Finally, the obtained supervisor consists of 12 control places to optimally control the net model. The designed control places are presented in Table 5, where the first

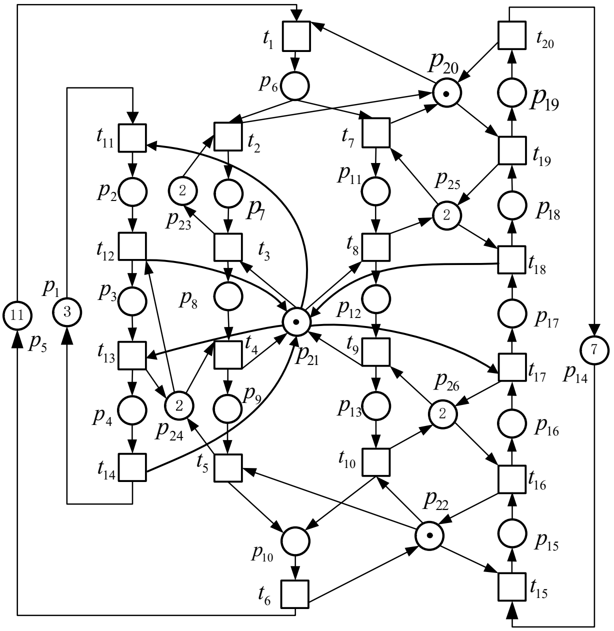

Figure 3. A net model from [23]

B	Include p_{s_i}	N_B Is Live?	Reachable Markings	DZ	LZ	p_{s_i}
1		Yes	17		17	
2		Yes	132		132	
3		No	637	5	632	$p_{s_1}p_{s_2}p_{s_3}p_{s_4}$
4	$p_{s_1}p_{s_2}\dots p_{s_4}$	No	2 106	2	2 104	$p_{s_5}p_{s_6}$
5	$p_{s_1}p_{s_2}\dots p_{s_6}$	No	5 192	2	5 190	p_{s_7}
6	$p_{s_1}p_{s_2}\dots p_{s_7}$	No	9 888	10	9 878	p_{s_8} p_{s_9}
7	$p_{s_1}p_{s_2}\dots p_{s_9}$	No	15 017	4	15 013	$p_{s_{10}}$
8	$p_{s_1}p_{s_2}\dots p_{s_{10}}$	Yes	18 972		18 972	
9	$p_{s_1}p_{s_2}\dots p_{s_{10}}$	Yes	20 980		20 980	
10	$p_{s_1}p_{s_2}\dots p_{s_{10}}$	No	21 536	11	21 525	$p_{s_{11}}p_{s_{12}}$
11	$p_{s_1}p_{s_2}\dots p_{s_{12}}$	Yes	21 581		21 581	

Table 3. The iteration steps for the net shown in Figure 3 by Algorithm 1

Parameters	[23]	[29]	[41]	[10]	The Work in [46]	Proposed Method
No. Monitors	18	6	19	5	17	12
No. Markings	6 287	6 287	21 562	21 581	21 581	21 581
Permissiveness (%)	29.13	29.13	99.91	100	100	100

Table 4. Performance comparison of control policies for the net shown in Figure 3

and second columns represent the index of the monitor and the PI I_i, respectively. The pre-transitions, post-transitions, and initial marking of control place p_{s_i} are given in the third to fifth columns, respectively.

i	I_i	${}^\bullet p_{s_i}$	$p_{s_i}^\bullet$	$M_0(p_{s_i})$
1	$\mu_{13} + \mu_{15} \leq 2$	t_{10}, t_{16}	t_9, t_{15}	2
2	$\mu_2 + \mu_8 + \mu_{15} \leq 2$	t_4, t_{13}	t_3, t_{11}	2
3	$\mu_{11} + \mu_{17} \leq 2$	t_8, t_{18}	t_7, t_{17}	2
4	$\mu_{12} + \mu_{16} \leq 2$	t_9, t_{17}	t_1, t_8, t_{16}	2
5	$\mu_{12} + \mu_{13} + \mu_{15} + \mu_{16} \leq 3$	t_{10}, t_{17}	t_8, t_{15}	3
6	$\mu_{11} + \mu_{16} \leq 3$	t_8, t_{17}	t_7, t_{16}	3
7	$2\mu_{11} + \mu_{12} + \mu_{13} + 2\mu_{15} + 2\mu_{16} \leq 8$	$t_8, t_{10}, 2t_{17}$	$2t_7, 2t_{15}$	8
8	$\mu_2+\mu_3+\mu_8+\mu_9+\mu_{13}+\mu_{15}+\mu_{16} \leq 5$	$t_5, t_{10}, t_{13}, t_{17}$	t_3, t_9, t_{11}, t_{15}	5
9	$\mu_6 + \mu_7 + \mu_{11} + \mu_{17} + \mu_{18} \leq 5$	t_3, t_8, t_{19}	t_1, t_{17}	5
10	$\mu_2+\mu_3+\mu_8+\mu_9+\mu_{11}+\mu_{15}+\mu_{16} \leq 6$	t_5, t_8, t_{13}, t_{17}	t_3, t_7, t_{11}, t_{15}	6
11	$\mu_6 + \mu_7 + \mu_9 + \mu_{11} + \mu_{13} + \mu_{15} + \mu_{16} + \mu_{17} \leq 9$	$t_3, t_5, t_8, t_{10}, t_{18}$	t_1, t_4, t_9, t_{15}	9
12	$2\mu_6+2\mu_7+2\mu_8+2\mu_9+2\mu_{11}+3\mu_{12}+3\mu_{13} + 4\mu_{15} + 4\mu_{16} + 2\mu_{18} \leq 24$	$2t_5, 3t_{10}, 4t_{17}, 2t_{19}$	$t_1, t_8, 4t_{15}, 2t_{18}$	24

Table 5. Control places computed for the net shown in Figure 3 by Algorithm 1

Next, a Petri net model of an FMS from [45] is considered, as shown in Figure 4. For this net model, we have $P^0 = \{p_{31}, p_{32}\}$ as idle places, $P_A = \{p_1 - p_{11}\}$ as operation places, and $P_R = \{p_{21}, p_{22}, p_{23}\}$ as resource places. It has 54 869 reachable markings with 51 506 markings in the LZ and 3 363 markings in the DZ. By applying the proposed method, the iteration processes are shown in Table 6.

Table 7 presents the performance comparison of different methods for this net model. This method can design an optimal supervisor with only five control places by taking 12 seconds. Therefore, the proposed approach obtains an optimal supervisor with a small number of monitors within less time, since the method in [10] costs more than four hours. Table 8 shows the details of designed control places.

Finally, another Petri net model of an FMS from [45] is selected to demonstrate the proposed method, as depicted in Figure 5. For this net model, we have the following place set partition: $P^0 = \{p_{11}, p_{17}, p_{18}\}$, $P_A = \{p_1 - p_{10}, p_{12} - p_{16}\}$, and $P_R = \{p_{22} - p_{27}\}$. It has 68 531 reachable markings of which 66 400 and 2 131 are legal and illegal markings, respectively. By using the proposed method, deadlocks are prevented in an iterative way. Table 9 shows the iteration processes of this net model.

Table 10 compares the performance of some approaches for this net model. Obviously, it indicates that the proposed method can design an optimal supervisor with a small number of control places, i.e., only eight control places are required. Similarly, by using this method, the optimal supervisor is obtained within nine

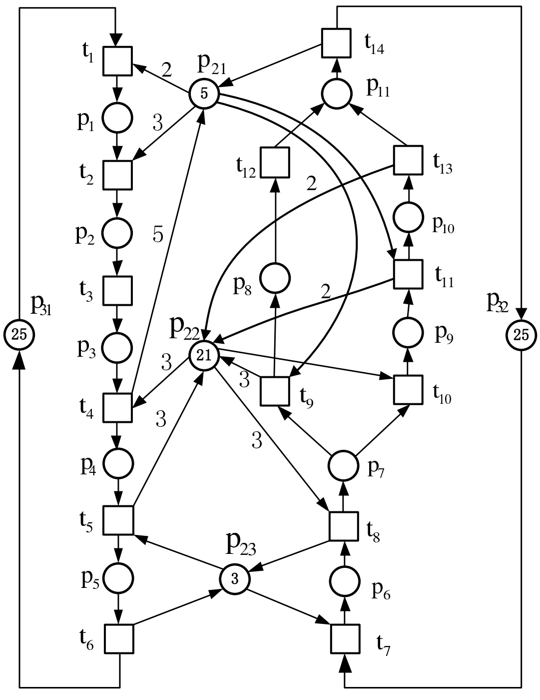

Figure 4. A Petri net model of an FMS from [45]

B	Include p_{s_i}	N_B Is Live?	Reachable Markings	DZ	LZ	p_{s_i}
1		Yes	12		12	
2		No	67	1	66	p_{s_1}
3	p_{s_1}	Yes	252		252	
4	p_{s_1}	Yes	767		767	
5	p_{s_1}	Yes	1 963		1 963	
6	p_{s_1}	No	4 366	4	4 362	p_{s_2}
7	$p_{s_1}p_{s_2}$	No	8 574	4	8 570	p_{s_3}
8	$p_{s_1}p_{s_2}p_{s_3}$	Yes	14 986		14 986	
9	$p_{s_1}p_{s_2}p_{s_3}$	No	23 404	2	23 402	p_{s_4}
10	$p_{s_1}p_{s_2}\dots p_{s_4}$	No	32 740	23	32 717	p_{s_5}
11	$p_{s_1}p_{s_2}\dots p_{s_5}$	Yes	41 162		41 162	
12	$p_{s_1}p_{s_2}\dots p_{s_5}$	Yes	47 203		47 203	
13	$p_{s_1}p_{s_2}\dots p_{s_5}$	Yes	50 363		50 363	
14	$p_{s_1}p_{s_2}\dots p_{s_5}$	Yes	51 380		51 380	
15	$p_{s_1}p_{s_2}\dots p_{s_5}$	Yes	51 506		51 506	

Table 6. The iteration steps for the net shown in Figure 4 by Algorithm 1

Parameters	[47]	TGAL in [44]	TGALW in [45]	[10]	The Work in [46]	Proposed Method
No. Monitors	7	5	8	2	10	5
No. Markings	51 386	48 752	51 548	51 506	51 506	51 506
Permissiveness (%)	99.76	94.65	99.83	100	100	100

Table 7. Performance comparison of control policies for the net shown in Figure 4

i	I_i	${}^\bullet p_{s_i}$	$p_{s_i}^\bullet$	$M_0(p_{s_i})$
1	$\mu_1 \leq 1$	t_2	t_1	1
2	$2\mu_2 + 2\mu_3 + \mu_7 + 2\mu_9 \leq 10$	$2t_4, t_9, 2t_{11}$	$2t_2, t_8, t_{10}$	10
3	$3\mu_2 + 3\mu_3 + 3\mu_7 + 4\mu_9 \leq 21$	$3t_4, 3t_9, 4t_{11}$	$3t_2, 3t_8, t_{10}$	21
4	$2\mu_2 + 2\mu_3 + \mu_4 + \mu_6 + 2\mu_9 \leq 13$	$t_4, t_5, t_8, 2t_{11}$	$2t_2, t_7, 2t_{10}$	13
5	$15\mu_2 + 15\mu_3 + 16\mu_4 + 16\mu_6 + 15\mu_7 + 21\mu_9 \leq 159$	$16t_5, t_8, 15t_9, 21t_{11}$	$15t_2, 5t_4, 16t_7, 6t_{10}$	159

Table 8. Control places computed for the net shown in Figure 4 by Algorithm 1

seconds, but the method in [10] takes more than three hours. The eight control places are given in Table 11.

5 CONCLUSIONS

This paper develops a deadlock prevention policy to design an optimal or near-optimal liveness-enforcing Petri net supervisor with a small number of monitors for FMSs. It prevents deadlocks in an iterative way by introducing a temporary GP. At each iteration, an ILPP (namely the MCPP) is formulated to compute a set of control places. These control places have the following characteristics:

B	Include p_{s_i}	N_B Is Live?	Reachable Markings	DZ	LZ	p_{s_i}
1		Yes	15		15	
2		Yes	117		117	
3		Yes	618		618	
4		No	2 398	1	2 397	p_{s_1}
5	p_{s_1}	No	7 138	3	7 135	p_{s_2}
6	$p_{s_1}p_{s_2}$	No	16 645	10	16 635	$p_{s_3}p_{s_4}$
7	$p_{s_1}p_{s_2}\cdots p_{s_4}$	No	30 890	11	30 879	$p_{s_5}p_{s_6}$
8	$p_{s_1}p_{s_2}\cdots p_{s_6}$	No	46 471	4	46 467	p_{s_7}
9	$p_{s_1}p_{s_2}\cdots p_{s_7}$	No	58 480	6	58 474	p_{s_8}
10	$p_{s_1}p_{s_2}\cdots p_{s_8}$	Yes	64 485		64 485	
11	$p_{s_1}p_{s_2}\cdots p_{s_8}$	Yes	66 181		66 181	
12	$p_{s_1}p_{s_2}\cdots p_{s_8}$	Yes	66 400		66 400	

Table 9. The iteration steps for the net shown in Figure 5 by Algorithm 1

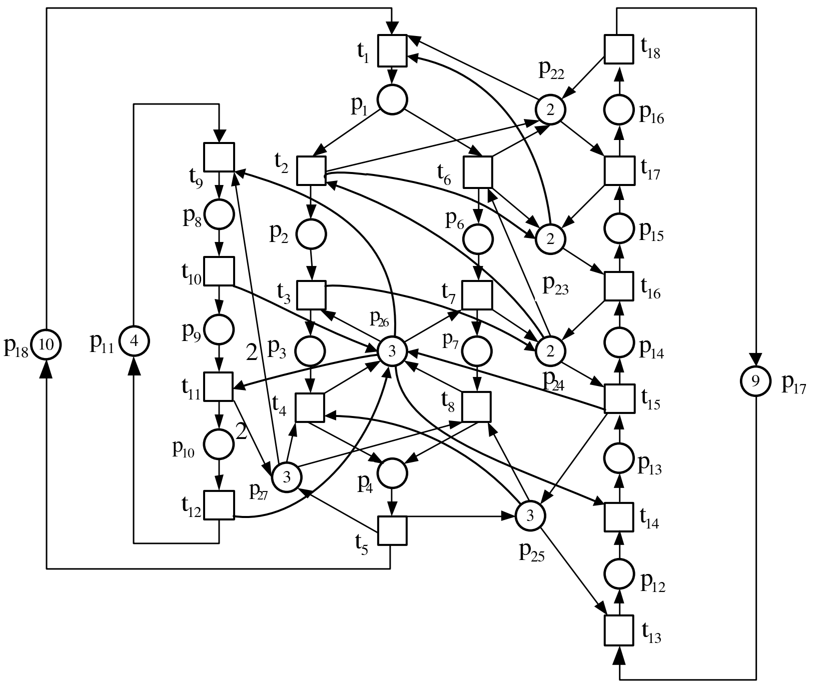

Figure 5. A net model of an FMS from [45]

Parameters	TGAL in [44]	TGALW in [45]	[10]	The Work in [46]	Proposed Method
No. Monitors	11	17	3	17	8
No. Markings	62 682	65 888	66 400	66 400	66 400
Permissiveness (%)	94.4	99.23	100	100	100

Table 10. Performance comparison of control policies for the net shown in Figure 5

i	I_i	${}^\bullet p_{s_i}$	$p_{s_i}^\bullet$	$M_0(p_{s_i})$
1	$\mu_1 + \mu_{14} \leq 3$	t_2, t_6, t_{16}	t_1, t_{15}	3
2	$\mu_2 + \mu_6 + \mu_{13} \leq 4$	t_3, t_7, t_{15}	t_2, t_6, t_{14}	4
3	$\mu_2 + \mu_3 + \mu_6 + \mu_7 + \mu_{12} + 2\mu_{13} \leq 7$	$t_4, t_8, 2t_{15}$	t_2, t_6, t_{13}, t_{14}	7
4	$\mu_3 + \mu_7 + \mu_{12} \leq 5$	t_4, t_8, t_{14}	t_3, t_7, t_{13}	5
5	$\mu_2 + \mu_3 + \mu_6 + \mu_7 + 2\mu_{12} \leq 9$	$t_4, t_8, 2t_{14}$	$t_2, t_6, 2t_{13}$	9
6	$\mu_1 + \mu_2 + \mu_6 + \mu_{13} + \mu_{14} \leq 6$	t_3, t_7, t_{16}	t_1, t_{14}	6
7	$\mu_1 + \mu_2 + \mu_3 + \mu_6 + \mu_7 + \mu_{12} + 2\mu_{13} + \mu_{14} \leq 9$	t_4, t_8, t_{15}, t_{16}	t_1, t_{13}, t_{14}	9
8	$\mu_1 + \mu_2 + \mu_3 + \mu_6 + \mu_7 + 2\mu_{12} + \mu_{14} \leq 11$	$t_4, t_8, 2t_{14}, t_{16}$	$t_1, 2t_{13}, t_{15}$	11

Table 11. Control places computed for the net shown in Figure 5 by Algorithm 1

1. they can prevent all illegal markings while all legal markings are reachable, i.e., they are maximally permissive; and
2. the number of control places is minimized and no redundant control place survives.

Compared with the previous work in [44], [45], and [46], the proposed method can design an optimal or near-optimal supervisor with fewer control places. Meanwhile, compared with the method in [10], it is more suitable for large-scale net models, since the number of constraints and variables in the MCPP is less at each iteration, and a supervisor with a small number of monitors is obtained in a reasonable time.

However, the proposed method also has some drawbacks. First, the design of supervisors needs to solve MCPPs, which is NP-hard. Second, compared with the work in [10], this method cannot ensure that an optimal supervisor with the minimal number of control places is designed. The reason is that some control places can be further compressed, but they are obtained in different iteration steps. Our future work will consider to minimize the number of control places. A possible way is that we can formulate a new ILPP after all legal and illegal markings are obtained.

Acknowledgements

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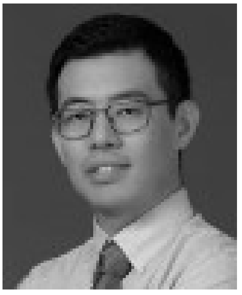

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