

## MODIFYING AND ACCELERATING THE METHOD OF MOMENTS CALCULATION

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**Abstract.** This manuscript deals with optimizing the numerical method called the method of moments (MoM). This method is widely utilized for field computation of 3D structures. MoM is exploited in hydraulics as well as in the electromagnetic field theory. Emphasis is put on minimizing calculations necessary for constructing a system of linear equations exploiting symmetry or similarity of elements of geometric structure. The manuscript also contains a comparison of computing times using standard MoM and a proposed modified MoM keeping the same structure of the solved construction.

**Keywords:** Numerical methods, method of moments, optimizing computation time

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## 1 INTRODUCTION

The evolution of computing technology is progressing hand in hand with the evolution of other natural, technical or economic sciences. Adequate powerful computing technology enables high precision analysis of process models, that more precisely describe the physical or social nature of things. Parallel computing or calculations performed on graphical processing units (GPU) facilitate solutions for computationally demanding tasks. Currently, due to these facts software based numerical simulations and modelling are enormously developing. On the one hand computing technology enables us to simulate models which are closer and closer to the objective reality of the simulated process, but on the other hand, users are increasingly tempted to use a “brute force” approach when simulating a complex process or a complex object. Brute force is most pronounced in increasing number of cores, memory capacity or computing power. In most cases attention is not paid to optimizing the model itself (number of segments, mirroring, repeating parts of model, etc.) or to optimizing the chosen numerical methods. This leads to constantly increasing requirements on computing hardware.

In this manuscript we present a specific numerical method – the method of moments and its optimization. In Section 2 the basic principles of MoM and the generation of a numerical model based on analytical description of a selected electromagnetic process are described. MoM is applied for computing simple conducting structures – dipole antenna. Section 3 deals with a model simulation consisting of various numbers of elements. This section also analyses the hardware and time requirements of the selected model. Section 4 describes the proposed modification of numerical method with a goal of minimizing computing time while keeping the number of segments unchanged. The numerical results are summarized in Section 5. Section 6 concludes the manuscript.

## 2 METHOD OF MOMENTS

This section describes the principles of MoM which will be used to analyze the electromagnetic (EM) field generated by a conductive structure in a free space. The presented analysis is based on [10]. In this particular case an EM field excited by a (point, line, planar, . . .) source is propagated in an environment (see Figure 1 b)).

The task is to find the configuration of field intensity in a free space environment and current distribution on the source surface. The starting point is Poisson equation:

$$\nabla^2 \mathbf{X} = \mathbf{Y} \quad (1)$$

where  $\mathbf{X}$  is the complex field distribution and  $\mathbf{Y}$  is the complex source function. We assume that the distribution of field sources is known. In case that this distribution is unknown but its integral magnitude in the closed volume  $V$  is known, then according

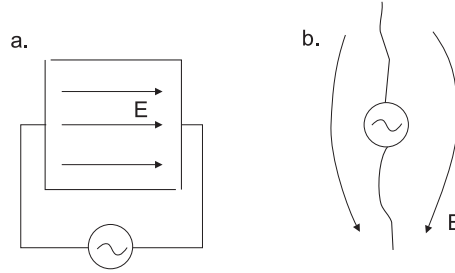


Figure 1. Field propagation: a) inside of a closed space, b) in a free space

to [15] we have to rewrite the Poisson equation to the form:

$$\nabla^2 \int_V \mathbf{J} \, dV = \mathbf{Y} \tag{2}$$

where  $\mathbf{J}$  is the searched complex function of electromagnetic field source distribution.

From (2) we calculate the unknown function  $\mathbf{J}$ , which in this case represents current distribution in a source field while  $\mathbf{Y}$  is powering this field. Solutions (2) often lead to an analytically unsolvable task. Therefore a numerical solution which gives very precise results is preferred. One of the most utilized numerical methods used to solve (2) is called the method of moments – MoM.

MoM is a numerical method suitable for solving integral equations often occurring in field theory (e.g. electromagnetic). This method is based on transforming the integral equation to a system of algebraic equations. This system can be formally described using a matrix equation, which is easily solvable using a computer.

Integral Equation (2) can be solved only if detailed parameters of the volume containing field sources are known. In Figure 2 two examples of geometries determining complex function (e.g. current distribution)  $\mathbf{J}$  are shown.

In the first case, (Figure 2a) the current distribution  $\mathbf{J}$  is located on the surface of the general volume  $V$ . It is clear that the current distribution is dependent on all coordinates of the used coordinate system and therefore it is not possible to simplify Equation (2). In the second case (Figure 2b)), the current distribution  $\mathbf{J}$  is located on the surface of the cylinder. Under assumption:  $d \rightarrow 0$  holds:

$$\mathbf{J}(x', y', z') \rightarrow \mathbf{J}(z') \tag{3}$$

for the integration area:

$$V \rightarrow \langle a, b \rangle. \tag{4}$$

The integral Equation (2) can be written symbolically taking into account [13] as:

$$\int_a^b \mathbf{f}(z, \xi) \, d\xi = \mathbf{g}(z) \tag{5}$$

where

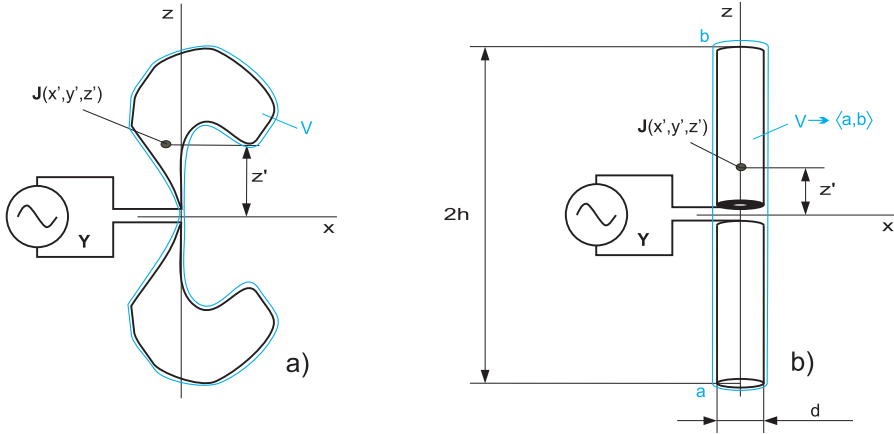


Figure 2. Determining integral boundary for volume V

- $\int_a^b$  is the linear operator  $L$  (in our case the integral),
- $a, b$  defines the analysed area (with respect to the choice of linear operator  $a, b$  are borders of the integral derived from the geometry of the analysed structure),
- $g$  is a known complex source function (supply voltage of the analysed structure),
- $f(z, \xi)$  is the unknown (searched) complex function (e.g. current distribution).

With the aid of MoM the unknown function is not searched for directly but it is approximated with a series with  $N$  basis functions:<sup>1</sup>

$$f \approx \tilde{f} = \alpha_1 f_1 + \alpha_2 f_2 + \dots + \alpha_N f_N = \sum_{n=1}^N \alpha_n f_n \tag{6}$$

where

- $\alpha_n$  are unknown complex coefficients,
- $f_n(z_0)$  are chosen known functions – basis functions.

This approximation can hold over the whole analyzed area  $a, b$ , or only on its part, the so called *segment*. The segment is a subset of the area  $a, b$ , where:  $J \doteq \text{const}$ . Certain programming skills and experience is necessary for selecting the length (shape) of the segment and also for choosing the basis functions. Suitable selection of these parameters can significantly decrease computing complexity, and thus the computing time.

MoM by contrast to the other numerical methods does not need to input border conditions. In this method the border conditions are directly contained in  $\int_V$ . Only

<sup>1</sup> Approximation is only formal, since coefficients  $\alpha_n$  are unknown.

the volume  $V$  contains field sources, which create radiated field in its vicinity (outside the volume  $V$ ).

By substituting (6) back into the solved Equation (5) and using linearity of integral it is possible to obtain:

$$\sum_{n=1}^N \alpha_n \int_a^b f_n(z, \xi) \, d\xi = \mathbf{g}(z) + R(z). \tag{7}$$

The solution acquired by MoM fulfills the border condition of Equation (5) only in discrete points. Between these points the border conditions are not fulfilled and discontinuity exists – residuum, in (7) denoted as  $R(z)$ . Discontinuity is caused by certain differences between the exact and approximate solution (denoted  $\tilde{f}$ ) of this equation.

For the most precise approximation of the solution residuum minimization is necessary. It can be obtained by the *method of weighted residua* [5]. This procedure introduces the term *weighted function*. It is defined so that its product with residuum  $R$ , integrated over the analysed area  $a, b$  is zero:<sup>2</sup>

$$\int_a^b w_m(z)R(z) \, dz = 0. \tag{8}$$

If for weighting we use  $N$  basis functions, a system of  $N$  equations with  $N$  unknown complex coefficients  $\alpha_n$  can be formed:

$$\sum_{n=1}^N \alpha_n \int_a^b w_m(z) \int_a^b f_n(z, \xi) \, d\xi \, dz = \int_a^b w_m(z)\mathbf{g}(z) \, dz, \tag{9}$$

respectively in matrix form:

$$\begin{pmatrix} \int_a^b w_1 \int_a^b f_1 & \dots & \int_a^b w_1 \int_a^b f_N \\ \vdots & \ddots & \vdots \\ \int_a^b w_N \int_a^b f_1 & \dots & \int_a^b w_N \int_a^b f_N \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} = \begin{pmatrix} \int_a^b w_1 \mathbf{g} \\ \vdots \\ \int_a^b w_N \mathbf{g} \end{pmatrix}. \tag{10}$$

In order the system (9) to be solvable on the interval  $a, b$  basis functions as well as weighting functions on this interval have to be independent. The careful reader would surely note that the solution of (5) depends primarily on the choice of basis functions. If those functions are properly chosen, the resulting system of equations (described by matrices) will be relatively small and the numerical solution will be very fast. It means that if we split the interval  $a, b$  into a small number of segments combined with a good choice of basis functions, the solution of the current distribution on a given interval will be relatively precise.

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<sup>2</sup> In the literature this product is also denoted is *inner product*

### 3 WAVE EQUATION SOLUTION

Based on the brief theoretical introduction to principles of MoM we can present an example of current distribution calculation at the dipole surface. The dipole is fed in its middle by a high frequency voltage source. In various literatures dealing with the theory of the electromagnetic field or antenna theory it is possible to find an analytical description of a dipole as radiating structure. In [7] is the description of a dipole created out of finite conductance material by means of the Pocklington equation:

$$\left(\frac{d^2}{dz^2} + k^2\right) \mathbf{A}_z(z) = \frac{jk^2}{\omega} (\mathbf{Z}(z)\mathbf{I}_z(z) - \mathbf{U}_A\delta(z)) \tag{11}$$

where  $\mathbf{A}_z(z)$  is a complex amplitude of vector potential (see [13]):

$$\mathbf{A}_z(z) = \frac{\mu}{4\pi} \int_{-h}^h \mathbf{I}_z(z') \frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}} dz', \tag{12}$$

- $\mathbf{I}(z)$  is searched function – the current distribution,
- $\mathbf{U}_A$  is supply voltage phasor,
- $k$  is wave number,  $k = 2\pi/\lambda$ ,
- $\omega$  is radian frequency of supply voltage,
- $\lambda$  is wave length.

Equation (12) is a special case of (1) respectively of (5), where the linear operator is represented with the integral  $\int_{-h}^h$  as well as with the second derivative  $\frac{d^2}{dz^2}$ . In order to apply MoM directly on Equation (12) it is necessary to modify MoM the following way:

- we substitute (15) into (12):

$$\left(\frac{d^2}{dz^2} + k^2\right) \frac{\mu}{4\pi} \int_{-h}^h \mathbf{I}_z(z') \frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}} dz' = \frac{jk^2}{\omega} (\mathbf{Z}(z)\mathbf{I}_z(z) - \mathbf{U}_A\delta(z)), \tag{13}$$

- the parenthesis with the second derivative is embedded into integral and the whole equation is multiplied by  $\frac{\omega}{jk^2}$ . After rearrangement we get:

$$\frac{j}{\omega\epsilon_0} \int_{-h}^h \left(\frac{d^2}{dz^2} + k^2\right) \mathbf{I}_z(z') \frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{4\pi\sqrt{(z-z')^2+a^2}} dz' + \mathbf{Z}(z)\mathbf{I}_z(z) = \mathbf{U}_A\delta(z). \tag{14}$$

Let us assume that the radiation structure has a cylindrical form, constructed from real material; it means it has a finite conductance. Therefore in Equation (11) the member  $\mathbf{Z}(z)$  – the impedance caused by dipole material resistivity also plays a role. In order to use MoM it is necessary to divide the dipole into separate

segments [11]. With a sufficient number of segments<sup>3</sup> it is possible to assume that the current distribution in each segment is constant and is equal to the current present in the middle of a dipole. In Figure 3 an example of dipole segmentation into 14 segments is shown.

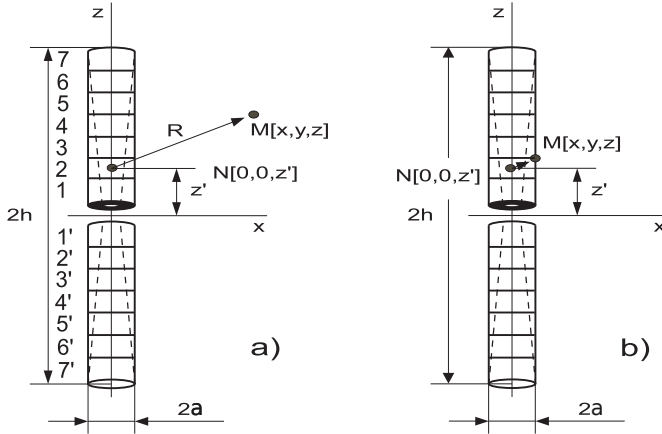


Figure 3. Splitting of a dipole into individual segments

Since the dipole is placed so that its axis is identical with the  $z$  axis and its center is in the origin of a coordinate system, by applying [5] we can assume that the same current distribution is in the segments 1 and 1', similarly in segments 2 and 2' and so on. Such geometry saves a lot of effort in the further analysis. Following on from Equation (11) we have to calculate the current  $I_z(z)$ , out of which we can later calculate the other antenna parameters. Based on MoM the integral equation must be rewritten into the form of a system of algebraic equations, which are easily solvable numerically or algebraically. In such a case the integral transforms to the sum. After rearrangement, Equation (14) has the form (10). In electrical engineering [3, 9] we can often encounter with the following form which is formally compliant with Ohms law:

$$\sum_{n=1}^N \mathbf{Z}_{mn} \mathbf{I}_n = \mathbf{U}_m \tag{15}$$

where

- $\mathbf{I}_n$  is column matrix of the unknown currents (corresponding with  $\alpha_n$  in (10)),
- $\mathbf{Z}_{mn}$  is the impedance matrix, representing the effect of the  $n^{\text{th}}$  segment on the observation point in the segment  $m$  (corresponding with double integral in (10)),
- $\mathbf{U}_m$  is the column matrix of supply voltages containing 1 in the segment position  $N/2$  and 0 in all other positions (corresponding with  $\int w_n \mathbf{g}$  in (10)).

<sup>3</sup> For a large number of segments, segment length approaches zero:  $\Delta l \rightarrow 0$ .

Impedance matrix  $\mathbf{Z}_{mn}$  is expressed the following way: position vector  $\vec{R}$  is defined from point  $N[0, 0, z' \equiv \xi]^4$ , lying in the center of a dipole to the observation point  $M[x, y, z]$ . If point  $M[x, y, z]$  is identified with the dipole surface, as is shown in Figure 3 b), then the magnitude of  $\vec{R}$  is equal to:

$$R_{z\xi} = \sqrt{(z - \xi)^2 + a^2} \tag{16}$$

The second derivation in (14) is applied to the integrand of a vector potential (12), which consequently can be expressed as:

$$\Xi_{z\xi} = \frac{d^2}{dz^2} \mathbf{G}_{z\xi} + k^2 \mathbf{G}_{z\xi} \tag{17}$$

where

$$\mathbf{G}_{z\xi} = \frac{e^{-jkR_{z\xi}}}{4\pi R_{z\xi}}. \tag{18}$$

Based on the previous equations it is possible to calculate elements of matrix  $\mathbf{Z}_{mn}$  as:

$$\frac{j}{\omega\epsilon_0} \begin{pmatrix} \int_{z_{1-1/2}}^{z_{1+1/2}} \Xi_{z_1\xi} d\xi + \mathbf{Z}_z^i(z_1)\Delta z_1 & \dots & \int_{z_{N-1/2}}^{z_{N+1/2}} \Xi_{z_1\xi} d\xi + \mathbf{Z}_z^i(z_1)\Delta z_1 \\ \vdots & \ddots & \vdots \\ \int_{z_{1-1/2}}^{z_{1+1/2}} \Xi_{z_N\xi} d\xi + \mathbf{Z}_z^i(z_N)\Delta z_N & \dots & \int_{z_{N-1/2}}^{z_{N+1/2}} \Xi_{z_N\xi} d\xi + \mathbf{Z}_z^i(z_N)\Delta z_N \end{pmatrix} \tag{19}$$

where

- $\mathbf{Z}_z^i(z_m)$  is the impedance per unit length in the center of the  $m^{\text{th}}$  segment,
- $z_n$  is the distance of the center of the  $n^{\text{th}}$  segment from the origin of the coordinate system,
- $z_{n+1/2}$  is the distance of the upper edge of the segment from the center of the coordinate system,
- $z_{n-1/2}$  is the distance of the lower edge of the segment from the center of the coordinate system,
- $\Delta z_n$  is the length of the segment  $n$ , where  $\Delta z_n = z_{n-1/2} - z_{n+1/2}$ .

In Equation (14) the value of the inner impedance per unit length still remains undefined. This value depends on the shape and material of the analysed structure. In order to continue in the previous analysis the course of the inner impedance can be chosen based on the assignment or we can use the mathematical expression for a concrete conductive geometry. In [16] the inner impedance is formulated for the analysed dipole:

$$\mathbf{Z}_z^i(z) = \frac{\mathbf{const}}{h - |z|}. \tag{20}$$

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<sup>4</sup> Coordinate  $z'$  we identify with  $\xi$ , in order to have the following formulas notated the same as in the previous section.



By substituting (20) into (19) we get an expression for the impedance matrix for the dipole shown in Figure 3. This impedance is substituted into the matrix Equation (15). Based on such input conditions and with the help of the solution of the system of linear equations we get column matrix  $\mathbf{I}_n$ , which represents the searched current distribution on the dipole surface.

#### 4 MOMENT METHOD MODIFICATION

As can be seen from the previous section, MoM is a very powerful tool for simulating conductive structures with infinite or finite inner conductivity. The weak point of this method is when solving structures in which the inner impedance changes depending on a coordinate. In such cases problems arise from time and hardware computing demands.

In the analysed case the inner impedance per unit length  $\mathbf{Z}_z^i(z_m)$  is approximated by a function e.g. (20). For  $z \rightarrow h$  this impedance approaches infinity. In such locations there is a significant difference between the impedance per unit length at the beginning  $\mathbf{Z}_z^i(z_{m-1/2})$  and at the end  $\mathbf{Z}_z^i(z_{m+1/2})$  of a particular segment. If the total impedance of a segment  $\mathbf{Z}^s(z_m)$  was calculated using mathematical simplification following from (19) it would introduce a significant error:

$$\mathbf{Z}^s(z_m) = \mathbf{Z}_z^i(z_m)(z_{m+1/2} - z_{m-1/2}). \quad (21)$$

In order to minimize this error it is necessary to increase the number of segments. But as the number of segments rises, so does the computing time. When solving a complex resistive structure with a similar expression of  $\mathbf{Z}_z^i(z_m)$  the above described way would unacceptably prolong computing time, which in turn will complicate any numerical analysis.

Based on these facts it is necessary to better describe the solved problem and at the same time to minimize the disadvantages inherent to MoM.

- For calculating a structure divided into  $N$  segments it is necessary to compute matrix  $\mathbf{Z}_{mn}$  containing  $N \times N$  complex numbers where each number has to be calculated using (19).
- By minimizing the number of segments errors in current distribution occur<sup>5</sup> and consequently errors occur also in calculation of other parameters of the analyzed structure.

Based on the previous analysis we end up with mutually exclusive requirements. If we want to speed up calculations we have to decrease the number of segments which results in inaccurate results. If we want to obtain accurate results we have to increase the number of segments. The possible solution to this problem stated in [6]

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<sup>5</sup> In (19) the inner impedance of a dipole per unit length  $\mathbf{Z}_z^i(z)$  is taken with respect to the center of a segment and is multiplied by the segment length. This simplification is suitable only for very short segments, otherwise an error occurs.

is to divide Equation (19) into two separate parts. The first part, containing integral and second order derivation is dependent on the shape of the analysed structure and structure of supply signal. The second part contains only the overall impedances  $\mathbf{Z}(z_m)$  of the individual segments. This part is dependant only on the shape and material of the dipole. By splitting Equation (19) the impedance matrix  $\mathbf{Z}_{mn}$  is also divided into two separate parts:

$$\mathbf{Z}_{mn} = \mathbf{Z}'_{mn} + \mathbf{Z}^s_{mn}. \tag{22}$$

The matrix  $\mathbf{Z}'_{mn}$  is identical to the impedance matrix contained in solution of the Pocklington Equation (11) using MoM for an infinitely conductive material. This fact implies that for computation of elements of matrix  $\mathbf{Z}'_{mn}$  we can assume an infinite conductance of the individual dipole segments.

Let's look closer at the creation of matrix  $\mathbf{Z}'_{mn}$ . To calculate its individual elements it is necessary to know only the magnitude of localization vector  $R_{z\xi}$  and not its direction. The element of the impedance matrix  $\mathbf{Z}'_{mn}$  from point 1 to 2 is the same as from point 2 to 1. This means:  $\mathbf{Z}'_{12} = \mathbf{Z}'_{21}$ . The similar statement holds true for the other segments as well. The magnitude of localization vector  $R_{z\xi}$  from point 1 to 1 is the same as from point 2 to 2, from 3 to 3 etc. (which represent a diagonal in matrix (23)). This means that the diagonal elements are equal. Based on these facts we can write the following equation:

$$\mathbf{Z}'_{mn} = \begin{pmatrix} \mathbf{Z}'_{11} & \mathbf{Z}'_{12} & \mathbf{Z}'_{13} & \mathbf{Z}'_{14} & \mathbf{Z}'_{15} & \dots \\ \mathbf{Z}'_{12} & \mathbf{Z}'_{11} & \mathbf{Z}'_{12} & \mathbf{Z}'_{13} & \mathbf{Z}'_{14} & \dots \\ \mathbf{Z}'_{13} & \mathbf{Z}'_{12} & \mathbf{Z}'_{11} & \mathbf{Z}'_{12} & \mathbf{Z}'_{13} & \dots \\ \mathbf{Z}'_{14} & \mathbf{Z}'_{13} & \mathbf{Z}'_{12} & \mathbf{Z}'_{11} & \mathbf{Z}'_{12} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{23}$$

From Equation (23) it follows that we do not have to solve  $m \times n$  numerical integrals and derivations but it is sufficient to calculate only the first row of the matrix and the other rows are combinations of the first one. This leads to the construction of Toeplitz matrix. When there are repeatable elements or parts of geometry the construction of Toeplitz matrix is relatively known. But only in the area of commercial generation of simulation programs. Using this simplification in the technical literature is quite rare.

In order not to generate a significant error in specifying the overall impedance of segments  $\mathbf{Z}^s(z_m)$  (elements of matrix  $\mathbf{Z}^s_{mn}$ ) it is necessary to modify their calculation so that real overall impedance of the individual segments can be calculated. In such a case it is not suitable to use an approximate impedance calculation (21) but an exact one with the aid of analytic integration:

$$\mathbf{Z}^s_{mn} = \int_{m-1/2}^{m+1/2} \frac{\mathbf{const}}{h - |z|} dz. \tag{24}$$

By integration of (24) we get:

$$\mathbf{Z}_{mn}^s = \begin{cases} \mathit{const} [\ln(h - z_{m-1/2}) - \ln(h - z_{m+1/2})], & z > 0, \\ \mathit{const} [\ln(h + z_{m-1/2}) - \ln(h + z_{m+1/2})], & z < 0. \end{cases} \quad (25)$$

By using (25) with regards to (19) and (22) it is possible to create a matrix of inner impedances:

$$\mathbf{Z}_{mn}^s = \begin{pmatrix} \mathbf{Z}_1^s & \mathbf{Z}_1^s & \mathbf{Z}_1^s & \mathbf{Z}_1^s & \dots \\ \mathbf{Z}_2^s & \mathbf{Z}_2^s & \mathbf{Z}_2^s & \mathbf{Z}_2^s & \dots \\ \mathbf{Z}_3^s & \mathbf{Z}_3^s & \mathbf{Z}_3^s & \mathbf{Z}_3^s & \dots \\ \mathbf{Z}_4^s & \mathbf{Z}_4^s & \mathbf{Z}_4^s & \mathbf{Z}_4^s & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (26)$$

By adding matrices (23) and (26) we will get back to classical MoM in which we have to solve a system of linear equations using (15).

## 5 NUMERICAL RESULTS

In the previous section we analysed the principles of MoM using a concrete example. We modified the method so the number of computations necessary to obtain the impedance matrix is minimized. At the same time by applying analytical integration we increased the precision of calculation of the overall segment impedance. This section describes the MoM implementation, analysis of the obtainable accuracy and computational speed. Cases of classical and modified MoM applied to dipoles from infinitely conducting and resistive materials are analysed as well.

MoM is implemented in the software environment *Mathematica* by means of functions applied in logical sequence of the individual steps. At first a Green function was created (18), where `distance[zm,zn,radius]` is the location vector  $R_{z\xi}$ :

```
greens[zm_, zn_,
a_] := E^(-I 2 \[Pi] distance[zm, zn, a])/(
4 Pi distance[zm, zn, a])
```

Next, by using a Green function an integrand of (17) without inner impedances per unit length was created:

```
kernel[zm_, zn_, a_] :=
D[greens[zm, zn, a], {zm, 2}] +
(2 Pi)^2 greens[zm, zn, a];
```

Let us assume for now the analysis of structures with infinite conductivity. Then for calculating the impedance matrix  $\mathbf{Z}_{mn} \equiv \mathbf{Z}'_{mn}$  we can use the function:

`impedanceskk [length, radius, N]`, the attributes of which are: dipole length, its radius and number of segments:

```

impedanceskk[length_, radius_, N_] :=
Module[{zm, zn, zmin=-length/2, delz = length/N, i, kern},
kern = kernel[zm, zn, radius];
Table[
zm = j;
Table[
-60.0 I NIntegrate[Evaluate[kern],
{zn, i - delz/2, i + delz/2}],
{i, zmin + delz/2, -zmin - delz/2, delz}
] // N, {j, zmin + delz/2, -zmin - delz/2, delz}
]]

```

To achieve quick convergence of calculations we introduced so called *MagneticFrill* matrix [1]. This matrix represents powering the dipole by a planar homogeneous wave – the so called source function. MagneticFrill is a matrix representing a narrow ring tightly fitting around dipole in its middle. The amplitude of source function has a Gaussian distribution with its mean placed in the centre of a dipole. The phase of the source function is approximately constant:

```

magneticfrill[length_, a_, b_, N_] :=
Module[{zmin=-length/2, zn, delz=length/N},
Table[2 Pi/Log[b/radius]*
(greens[i, 0, a] - greens[i, 0, b]),
{i, zmin + delz/2, -zmin - delz/2, delz}
] // N
]

```

Calculation of currents matrix (current distribution) is possible using the following formula:

```

currents = LinearSolve[
impedanceskk[1 0.047, 0.0005, 101],
-magneticfrill[1 0.047, 0.0005, 2.23 * 0.0005, 101]
];

```

All data in the arguments of functions are related to wave length of the incident wave. In this case the dipole length is  $2h = 0.047\lambda$  and its radius is  $d = 0.0005\lambda$ . The dipole is divided into  $N = 101$  segments. In the function MagneticFrill we use the argument  $2.23 \times 0.0005$ , which represents the width of the source function ring. This value was chosen based on [14].

With the aid of the presented mathematical background and *Mathematica* it is possible to calculate an impedance matrix  $\mathbf{Z}_{mn}$  for different number of segments. Computing time was measured using the standard Mathematica command. Calculations were performed using one core of an Intel Core i7 CPU 860@2.8GHz processor. The dependence of computing time on number of segments is shown in Figure 4. The number of segments varied from 21 to 101, which resulted in

a quadratic increase of computing time from 6 to 147 [s]. Such a quadratic increase of computation time was expected due to the quadratic increase of segment numbers in the impedance matrix. The measured total execution time is relatively high. From [2] follows that MoM gives quite accurate results if segment length is much less than physical length of dipole ( $l_s \ll 2h$ ). At the same time the segment length has to be at least two times its radius ( $l_s > 2d$ ). In further analysis the number of segments will vary by around 50. With this value the total execution time is 38 [s], which is with respect to the geometric simplicity of the dipole and available computing power, an enormously high value.

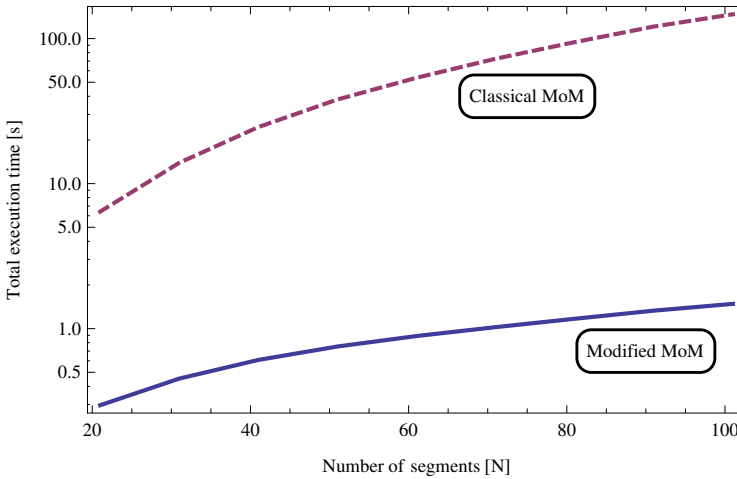


Figure 4. Total execution time for different numbers of segments

Let us perform a computation of the impedance matrix  $\mathbf{Z}_{mn}$  using simplification (23). In this case only one row of matrix  $\mathbf{Z}_{mn}$  is calculated the following way:

```

impedances[length_, a_, N_] :=
Module[{zm, zn, zmin=-length/2, delz = length/N, i, kern},
kern = kernel[zm, zn, a];
zm = zmin + delz/2;
Table[
-60.0 I NIntegrate[Evaluate[kern],
{zn, i - delz/2, i + delz/2}],
{i, zmin + delz/2, -zmin - delz/2, delz}
] // N
]

```

The other rows are obtained from combining the first row using the Toeplitz matrix. The current matrix is then calculated the following way:

```
currents = LinearSolve[
ToeplitzMatrix[impedances[1 0.047, 0.0005, 51]],
-magneticfrill[1 0.047, 0.0005, 2.23 * 0.0005, 51]
];
```

The current distribution on the dipole is calculated using the modified MoM by applying (23). The input parameters (dipole length, its radius and number of segments) are the same as in the previous case. The plot of total execution time based on the number of segments is shown in Figure 4, its magnified version is in Figure 5. From Figure 5 it is possible to read that the total execution time for 50 segments has been shortened to 0.7 [s]. At the same time the quadratic dependence of total execution time on number of segments has changed to linear. The improved total execution time represents speeding up the computation by 60× compared to the conventional method. This time reduction is very important especially for very precise calculation of input impedances for complex, geometrically repeating structures. It is necessary to note that depicting and searching the impedance matrix offers a solution by using the Toeplitz matrix.

For 50 segments memory usage was evaluated as well. Since the computation was processed in system *Mathematica*, the most of the system memory was used by it. To start *Mathematica* kernel approximately 28.8MB of memory is required. The calculation of current matrix with classical MoM took another – approximately 8.0 MB. The same calculation performed with the modified MoM took approximately 4.98 MB – that represents approximately 38 % memory saving.

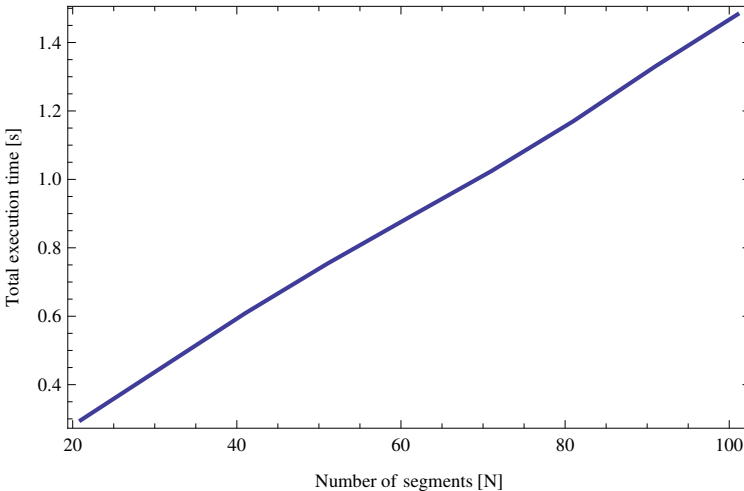


Figure 5. Total execution time for different numbers of segments obtained with modified MoM

### 5.1 Example of Resistive Structure Calculation

Sometimes a resistive structure depicting the matrix  $\mathbf{Z}_{mn} \neq \mathbf{Z}'_{mn}$  will not reveal any regularities or repetition of segments. Even in the case of geometrically repeating structure it is not clear how to simplify and speed up computations.

At first let us calculate the resistive structure – resistive dipole using classical MoM. The starting point is (19) and inner impedance per unit length  $\mathbf{Z}'_z(z)$  is taken from [4]. Calculations can be performed using the following Mathematica code:

```
impedanceskkZi[Zi_, length_, radius_, N_] :=
Module[{zm, zn, zmin=-length/2, delz = length/N, i, kern},
kern = kernel[zm, zn, radius];
Table[
zm = j;
Table[
-(60.0 I NIntegrate[Evaluate[kern],
{zn, i - delz/2, i + delz/2}] + (
60 Zi)/(-zmin - Abs[j]) delz),
{i, zmin + delz/2, -zmin - delz/2, delz}
] // N, {j, zmin + delz/2, -zmin - delz/2, delz}
]]
```

Similarly as in previous cases we calculate the elements of impedance matrix for the same dipole parameters like at the beginning of the presented analysis. The graph of total execution time based on the number of segments is shown in Figure 6.

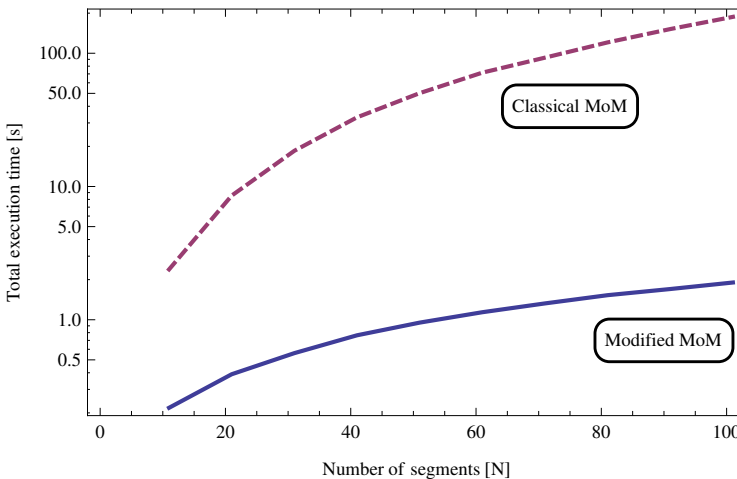


Figure 6. Total execution time for different number of segments in case of resistive structure

By comparing the dashed lines in Figure 4 and Figure 6 (representing using classical MoM) it can be observed that the shape of both graphs is similar. In the case of resistive dipole total execution time has increased. For 50 segments it is approximately 50 [s], which represents 30 % increase of computing time with respect to the solution of classical dipole.

Let us apply the theoretical analysis described in (22) and (24)–(26) for solving a resistive structure. The first row of matrix  $Z'_{mn}$  is calculated using the function onlyZi[Zi, length, radius, N]:

```
onlyZi[Zi_, length_, radius_, N_] :=
Module[{zm, zn, zmin=-length/2, delz = length/N, i, kern},
kern = kernel[zm, zn, radius];
Table[
zm = j;
Table[
(60 Zi)/(-zmin - Abs[j]) delz,
{i, zmin + delz/2, -zmin - delz/2, delz}
] // N, {j, zmin + delz/2, -zmin - delz/2, delz}
]]
```

The loading of all rows and columns of the impedance matrix  $Z_{mn}$  in the case of resistive dipole (dipole length  $2h = 0.047\lambda$ , its radius  $d = 0.0005\lambda$  and number of segments  $N = 101$ ) is obtained using the following commands:

```
ToeplitzMatrixB[impedances[0.047, 0.0005, 101]] -
onlyZi[8.10988, 0.047, 0.0005, 101]
```

The plot of total execution time versus number of segments is in Figure 6 (solid line) and its magnified version is in Figure 7.

Similarly as in Figure 5 we can observe linear dependency of total execution time on the number of segments. Since it was necessary to compute two impedance submatrices, the computing time rose by 10 % compared to the structure with infinite conductivity. If we compare the computing time for an impedance matrix of resistive structure using classical and modified MoM it can be observed that computing time has decreased more than  $50\times$  for 50 segments.

Similarly as in the previous case we were analyzing the memory usage for calculation of the current matrix for 50 segments. Calculation of this matrix using the classical MoM took approximately 8.0 MB of memory. The same calculation using modified MoM required approximately 5.22 MB – it represents approximately 35 % of memory savings compared to the classical method.

## 6 CONCLUSIONS

By comparing computing times for solving a dipole with infinite conductivity using classical and modified method it can be observed that the proposed modified method



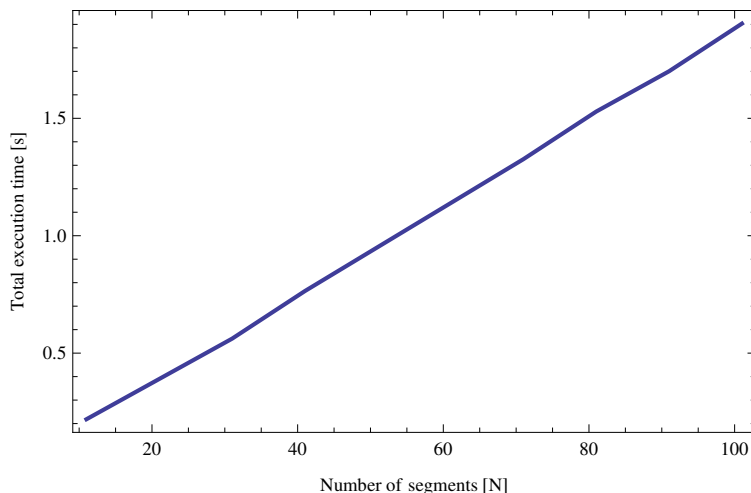


Figure 7. Total execution time for different number of segments in the case of resistive structure obtained with modified MoM

was able to create an impedance matrix  $60\times$  faster. In the case of the resistive dipole creation of the impedance matrix was  $50\times$  faster than using the classical MoM.

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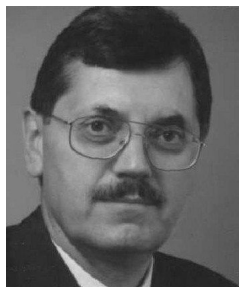
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