IRFUM: IMAGE RETRIEVAL VIA FUZZY MODELING

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Abstract. To reduce the semantic gap in the content based image retrieval (CBIR) systems we propose a fuzzy rule base approach. By submitting a query to the proposed system, it first extracts its low-level features and then checks its rule base for determining the proper weight vector for its distance measure. It then uses this weight vector to determine what images are more similar to the query image. For the training purpose, an algorithm is provided by which the system adjusts its fuzzy rules' parameters by gathering the trainers' opinions on which and how much the image pairs are relevant. For further improving the performance of the system, a feature space dimensionality reduction method is also proposed. We compared the proposed method with some other common ones. Our experiments on a subset of the Corel database containing 59600 images show that the proposed method is more precise than these compared methods based on the precision and recall criterions.

Keywords: Content based image retrieval, Fuzzy rule base, Fuzzy modeling
1 INTRODUCTION

One of the most demanding requirements of multimedia systems is to be able to search, browse and retrieve images, audios and videos. This functionality can be achieved only if all of such files are annotated with keywords representing semantic concepts, and this is only realizable by expert human annotators. However, manual annotation is not only slow but also extremely expensive, rendering such a process unfeasible in practical applications. This was the main motivation for a large volume of studies to design an automatic system that can retrieve multimedia without the need for the manual annotations. Unfortunately, the gap between the capabilities of current image understanding algorithms and the richness and subjectivity of semantics in human interpretations of audiovisual media is a formidable hurdle in the way to achieve this objective. To tackle the underlying problems many approaches have been proposed over the past few years. The most promising ones endeavor to merge various low-level content representations and learning approaches in an attempt to simulate human inference and reasoning capabilities. Here, low-level descriptors are commonly used to infer semantic meaning [1–4]. Since the subjectivity and fuzziness of human reasoning adds an unavoidable noise component to the actual image understanding problem, some kind of learning from the experts needs to be considered as well. Indeed, the idea of simulating human understanding is also related to such learning. It can be argued that incorporating users’ knowledge and preferences into a learning approach could eventually bring the machine to “think” as a human.

One can find fuzzy-based image retrieval methods in the literature. For addressing the irrelevant visual contents, in [5], a probabilistic fuzzy region matching algorithm has been adopted to retrieve and match images precisely at object level, which copes with the problem of inaccurate segmentation.

In [6], the authors propose a classification method, based on the dominant color(s) of the images. The process consists of two steps: first, assigning a colorimetric profile to the image in HLS space (Hue, Lightness, Saturation) and then, handling the query for the retrieval. To achieve the first step, the definition of hue is done using a fuzzy representation that takes into account the nonuniformity of color distribution. Then, lightness and saturation are represented through linguistic qualifiers also defined in a fuzzy way. Finally, the profile is built through fuzzy functions representing the membership degree of the image to different classes. Thus, the query for image retrieval is a pair (hue, qualifier). The second step looks for a match between the query and the profiles. In order to improve the software and to make it more flexible, the user can re-define the fuzzy representation of Hue, Lightness and Saturation, according to his own perception.

In [7], a fuzzy relevance feedback approach is proposed which enables the user to make a fuzzy judgement. It integrates the users’ fuzzy interpretation of visual content into the notion of relevance feedback. A learning approach is proposed using a fuzzy radial basis function network. The network is constructed based on the users’ feedbacks.
Some more related works on using fuzzy logic for image retrieval can be found in [8–15]. In the above methods, the authors use fuzzy logic to deal with the vagueness and ambiguity in some parts of their image retrieval system. However, none of the above-mentioned approaches try to provide a rule-based system for simulating human operation in image retrieval task, which is the most important property of our proposed method.

In this paper, we try to model the operation of an expert based on the input-output data he or she use in retrieval of the images, using fuzzy rules created by an automatic process. This method is called IRFuM, which stands for Image Retrieval via Fuzzy Modeling. Fuzzy modeling is widely used for modeling a control process to build stable controllers. For some of the most popular methods refer to [16–20].

Modeling the human operation can be done by many other techniques, but fuzzy modeling simplifies analysis and design of the system, since fuzzy rules are able to simulate our inference system. We propose a fuzzy model that upon receiving a query image assigns different weights for image regions and features during the retrieval process. The system learns these weights from the expert users’ input-output data. This can reduce the semantic gap in CBIR systems, as our experiments confirm this fact.

Some parts of our proposed method are reported distinctly in [21,22]. In [23,24], we proposed two other approaches in semantic based image retrieval using fuzzy modeling. The main difference between our proposed approach here and those approaches is in estimation of FE weights.

The rest of this paper is organized as follows. In Section 2, we present our method for reducing the semantic gap in image retrieval systems. The details of the proposed fuzzy modeling and fuzzy system are described in this section. Section 3 is dedicated to the experimentations and results and finally, Section 4 concludes the paper.

The notations of the variables are presented in Table 1.

2 FUZZY-BASED MODELING OF HUMAN OPERATION

Fuzzy modeling is an efficient way for modeling the human behavior to solve a problem. In this section, we first describe the operation of the designed fuzzy system and then give the details of the training algorithm. Our proposed model requires an offline training phase for which we propose a complete training algorithm for reduction of feature space dimensionality, determining the number of fuzzy rules, and adjusting the fuzzy set parameters. This training algorithm leads to a fuzzy system whose we describe the operation at run-time in the following sub-section.

2.1 Fuzzy System for Image Retrieval: Run-Time Operation

The overall structure of the proposed fuzzy system is depicted in Figure 1. When an image is given to the system, it first extracts predefined features of the image.
<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ij}$</td>
<td>Trapezoidal shaped fuzzy set of the $i^{th}$ dimension for the $j^{th}$ rule at the premise part</td>
</tr>
<tr>
<td>$B_{ij}$</td>
<td>Trapezoidal shaped fuzzy set of the $i^{th}$ dimension for the $j^{th}$ rule at the consequent part</td>
</tr>
<tr>
<td>$C$</td>
<td>The number of classes for the classification based methods</td>
</tr>
<tr>
<td>$c$</td>
<td>The number of clusters</td>
</tr>
<tr>
<td>$C^j$</td>
<td>The $j^{th}$ cluster produced by the clustering module</td>
</tr>
<tr>
<td>$D_{ij}$</td>
<td>Weighted Euclidean distance between a pair of query and target images</td>
</tr>
<tr>
<td>$E$</td>
<td>Mean square error in the defined Lagrange minimization problem of Equation (11)</td>
</tr>
<tr>
<td>$F$</td>
<td>Number of user feedbacks</td>
</tr>
<tr>
<td>$F(.)$</td>
<td>A function for converting similarity score vector $S$ to the appropriate weight vector $W$ based on the trainers’ intension</td>
</tr>
<tr>
<td>$F^j$</td>
<td>$M \times N$ matrix of feature vectors of all target images</td>
</tr>
<tr>
<td>$f_{it}$</td>
<td>The $i^{th}$ feature element of the $t^{th}$ target image, $1 \leq t \leq N$</td>
</tr>
<tr>
<td>$F^q$</td>
<td>$M \times 1$ feature vector of the query image</td>
</tr>
<tr>
<td>$f_{iq}$</td>
<td>The $i^{th}$ feature element of the query image, $1 \leq t \leq N$</td>
</tr>
<tr>
<td>$I_{j_{\text{max}}}$</td>
<td>The maximum value of the $i^{th}$ feature element of all points in the $j^{th}$ cluster</td>
</tr>
<tr>
<td>$I_{j_{\text{min}}}$</td>
<td>The minimum value of the $i^{th}$ feature element of all points in the $j^{th}$ cluster</td>
</tr>
<tr>
<td>$g(.)$</td>
<td>A function for converting distance values to the similarity scores</td>
</tr>
<tr>
<td>$i_{\text{kn}}$</td>
<td>$N \times 1$ vector of elements $1$</td>
</tr>
<tr>
<td>$M$</td>
<td>Feature space dimensionality</td>
</tr>
<tr>
<td>$m$</td>
<td>Reduced dimensionality</td>
</tr>
<tr>
<td>$N$</td>
<td>The number of images in the database</td>
</tr>
<tr>
<td>$N_r$</td>
<td>The number of fuzzy rules</td>
</tr>
<tr>
<td>$P$</td>
<td>The average precision of the returned images for all query images</td>
</tr>
<tr>
<td>$P_{\text{max}}$</td>
<td>Maximum average precision used in Algorithm 2</td>
</tr>
<tr>
<td>$P_{\text{max,prev}}$</td>
<td>$P_{\text{max}}$ of the previous stage used in Algorithm 2</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>The number of returned images defined for the image retrieval system</td>
</tr>
<tr>
<td>$\mathcal{R}_f$</td>
<td>Set of all remained FEs used in Algorithm 2</td>
</tr>
<tr>
<td>$S$</td>
<td>$N \times 1$ score vector of elements $s_i$ assigned to each of $N$ images in the training set for a certain query</td>
</tr>
<tr>
<td>$S(.)$</td>
<td>The criterion for evaluating a certain clustering</td>
</tr>
<tr>
<td>$S_f$</td>
<td>Set of all selected FEs used in Algorithm 2</td>
</tr>
<tr>
<td>$v_i$</td>
<td>Vector expressing the center of $i^{th}$ cluster</td>
</tr>
<tr>
<td>$W$</td>
<td>$M \times 1$ weight vector</td>
</tr>
<tr>
<td>$\mathcal{W}$</td>
<td>Number of weights used for the RBF network for FRBF method [7]</td>
</tr>
<tr>
<td>$w_i$</td>
<td>Weight of the $i^{th}$ feature element, $0 \leq i \leq M$</td>
</tr>
<tr>
<td>$\hat{w}_i$</td>
<td>Inferred weight value for the $i^{th}$ dimension</td>
</tr>
<tr>
<td>$w^*_{ij}$</td>
<td>Defuzzified version of the $i^{th}$ fuzzy set in the consequent part of the $j^{th}$ rule</td>
</tr>
<tr>
<td>$X$</td>
<td>$M \times 1$ vector of summation of feature differences weighted by the scores</td>
</tr>
<tr>
<td>$Y$</td>
<td>$M \times M$ matrix of all Euclidian distances</td>
</tr>
<tr>
<td>$Z$</td>
<td>$N \times M$ matrix of feature element differences between a certain query image and all the database images</td>
</tr>
<tr>
<td>$\mu'$</td>
<td>The degree of match for the premise part of the $j^{th}$ rule</td>
</tr>
</tbody>
</table>

Table 1. Nomenclature
Fig. 1. Overall structure of the proposed fuzzy system for image retrieval

The resultant feature vector is used as an input to the fuzzy inference module which creates \( M \) weights \( w_1, w_2, \ldots, w_M \), where \( M \) denotes the dimension of the feature space and \( \sum_{i=1}^{M} w_i = 1 \). These weights along with the features of the query image are used for computing the distance measure as follows:

\[
D_{qt} = \sum_{i=1}^{M} w_i |f^q_i - f^t_i|
\]  

(1)

in which \( f^q_i \) and \( f^t_i \) denote the \( i^{th} \) FE of query and target images, respectively. We assume that the FEs are normalized to lie between 0 and 1. \(|.|\) measures the absolute value of its operand. \( f^t_i \)'s are stored for each database image as shown in Figure 1.

Note that we could assign a single weight to a group of FEs instead of individually assigning different weights. For example, it is possible to assign a single weight to all shape FEs, a single weight to all color FEs, etc. This is appropriate when the training set is small. However, the more weight values are used, the more precise system is resulted. Hence, the proposed system has the flexibility in assigning weight values for a group of FEs or individually to each FE. Herefrom, we assume that each FE takes a separate weight value. However, extension to the case of assigning a single weight to a group of FEs is straightforward.

The fuzzy rule base contains If-Then rules of the following form:

**Rule 1:** if \( f^q_1 \) is \( A^1_1 \) and \( f^q_2 \) is \( A^1_2 \) and \( f^q_M \) is \( A^1_M \), then \( \omega_1 \) is \( B^1_1 \) and \( \omega_2 \) is \( B^1_2 \) and \( \ldots \omega_M \) is \( B^1_M \).
Rule 2: if $f_{q1}^i$ is $A_{21}^i$ and $f_{q2}^i$ is $A_{22}^i$ and ... $f_{qM}^i$ is $A_{2M}^i$, then $\omega_1$ is $B_{11}^i$ and $\omega_2$ is $B_{12}^i$ and ... $\omega_M$ is $B_{1M}^i$.

Rule $N_r$: if $f_{q1}^i$ is $A_{N_r1}^i$ and $f_{q2}^i$ is $A_{N_r2}^i$ and ... $f_{qM}^i$ is $A_{N_rM}^i$, then $\omega_1$ is $B_{1N_r}^i$ and $\omega_2$ is $B_{2N_r}^i$ and ... $\omega_M$ is $B_{MN_r}^i$.

in which $A_{ji}$ and $B_{ji}$ are trapezoidal-shaped fuzzy sets and $N_r$ denotes the number of rules. Given the input feature vector $F^q = [f_{q1}, f_{q2}, \ldots, f_{qM}]^T$, the fuzzy inference module in Figure 1 performs the following steps:

1. Calculates the degree of match, $\mu^j$, in the premises for the $j^{th}$ rule, $1 \leq j \leq N_r$, as:
   \[ \mu^j = \prod_{i=1}^{M} A_{ji}^j(f_{qi}^i) \]  
   where $A_{ji}(.)$ computes the membership value of its operand.

2. Defuzzifies $B_{ji}$'s in the consequents using any defuzzification method such as taking the center of gravity:
   \[ w_{ji} = \frac{\int B_{ji}^j(f) f \, df}{\int B_{ji}^j(f) \, df}. \]

3. Computes the inferred weight values, $\hat{w}_i$, by taking the weighted average of $w_{ji}$'s with respect to $\mu^j$
   \[ \hat{w}_i = \frac{\sum_{j=1}^{N_r} \mu^j w_{ji}^j}{\sum_{j=1}^{N_r} \mu^j}. \]

These $\hat{w}_i$'s along with feature vector $F^q$ are used for computation of distances $D_{qi}$ of the query image from target images in the image database using Equation (1). Then, the system returns images with distance $D_{qi}$ less than a predefined threshold $T$.

### 2.2 Training the Fuzzy System: Off-Line Phase

In making a fuzzy model, four steps should be considered:

1. Designing the format of fuzzy rules.
2. Determining the relevant inputs.
3. Determining the number of rules.
4. Calculating the parameters.

To generate the input-output dataset, a subset of $N$ images has been used among them and $N_q$ query images have been selected randomly. An expert looks at each query image and gives an score from 0 to 5 to each database image indicating how much it is similar to the query image. Table 2 shows the scores for different
5: completely similar  
4: very similar  
3: similar  
2: somewhat similar  
1: not similar but relevant in some sense  
0: not similar and not relevant

Table 2. Scores of similarity for different situations

Algorithm 1: Training the fuzzy system from input-output data

input: The set of training images and their relationships  
output: The set of fuzzy rules and their parameters

1 Reduce the feature space dimensionality (Section 2.2.1);  
2 Calculate matrix \( \mathbf{W} \) (Section 2.2.2);  
3 \( c \leftarrow 1; // \) Number of clusters  
4 \( S(1) \leftarrow +\infty; \)
5 repeat  
6 \( c \leftarrow c + 1; \)
7 Run fuzzy clustering using the FCM algorithm;  
8 Compute the clustering performance criterion: \( S(c) \) (Equation (5));  
9 until \( S(c) > S(c - 1); \)
10 Find the parameters of the fuzzy sets at the consequence part (Section 2.2.3);  
11 Find the parameters of the fuzzy sets at the premise part (Section 2.2.3);  
12 Adjust the parameters of the fuzzy sets at the premise part (Section 2.2.4);
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not take an ordinary fuzzy partition of the input space, since this can increase the number of rules exponentially with the number of FEs. We use the fuzzy c-means (FCM) method, to determine the number of fuzzy rules, by clustering the output data (computed weights).

The number of clusters is determined using the criterion \( S(c) \) defined as

\[
S(c) = \sum_{j=1}^{n} \sum_{i=1}^{c} \left( ||w_j - v_i||^2 - ||v_i - \bar{w}||^2 \right),
\]

(5)

where:

- \( n \): the number of data to be clustered;
- \( c \): the number of clusters, \( c \geq 2 \);
- \( w_j \): the \( j^{th} \) weight vector;
- \( \bar{w} \): average of data: \( w_1, w_2, \ldots, w_n \);
- \( v_i \): vector expressing the center of \( i^{th} \);
- \( ||.|| \): norm.

The number of clusters, \( c \), is determined so that \( S(c) \) reaches a minimum as \( c \) increases: it is supposed to be a local minimum as usual. As can be seen in Equation (5), the first term of the right-hand side is the variance of the data in a cluster and the second term is that of the clusters themselves. Therefore the optimal clustering is considered to minimize the variance in each cluster and to maximize the variance between the clusters.

2.2.1 Feature Space Dimensionality Reduction

Algorithm 2 is proposed for feature space dimensionality reduction. We are looking for a subset of FEs, \( F \), that maximizes the precision of the retrieval process. Therefore, the total number of cases equal to the number of subsets except an empty subset of \( F \), i.e., \( 2^M - 1 \), where \( M \) indicates the dimensionality of \( F \). Here we use a heuristic method to select some FEs amongst the total set of FEs; we increase the number of FEs one by one, evaluating a criterion. Our algorithm checks at most \( M(M - 1)/2 \) subsets to find optimal subset of FEs.

First, we begin with a CBIR system with one input. In this case, we compute the \( L^1 \) distance measure of each test query image feature vector from that of database images. This distance measure is defined as follows:

\[
D_{CBIR}^{gb} = \sum_{i=1}^{M} |f_q^i - f_t^i|.
\]

(6)

We assume that the FEs are normalized to lie between 0 and 1. These distances should be converted to similarity scores. Any decreasing function \( g(D) \) with the
**Algorithm 2:** Reduction of feature space dimensionality.

**input**: The set of all FEs

**output**: A subset of FEs

1. \( R_f \leftarrow \{1, 2, \ldots, M\} \); // Set of all remained FEs
2. \( S_f \leftarrow \emptyset \); // Set of all selected FEs
3. \( P_{\text{max}} \leftarrow -\infty \); // Maximum average precision
4. \( P_{\text{max,prev}} \leftarrow -\infty \); // \( P_{\text{max}} \) of the previous stage
5. \( P(0) \leftarrow -\infty \);
6. **repeat**
    7. **repeat**
       8. Choose next FE from \( R_f \);
       9. Add the selected FE to \( S_f \);
       10. Compute the distance of each test query image feature vector from that of each test database image using only FEs from \( S_f \);
       11. Compute the average precision \( P \) (Equation (9));
       12. **if** \( P > P_{\text{max}} \) **then**
           13. \( P_{\text{max}} \leftarrow P \);
           14. \( f_{\text{max}} \leftarrow \text{Currently Selected FE} \);
       **end**
       16. **if** \( P < P_{\text{max,prev}} \) **then**
           17. Remove currently selected FE from \( R_f \);
       **end**
       19. Remove currently selected FE from \( S_f \);
    **until** all elements of \( R_f \) are being selected;
20. **until** \( R_f = \emptyset \);

Following constraints can be chosen:

\[
\begin{align*}
g(0) &= 5, \\
g(1) &= 0. \\
\end{align*}
\]

(7)

We use the following empirical function:

\[
g(D) = 5(1 - D). \tag{8}
\]

After finding the similarity scores, the average precision, \( P \), for all test query images for each number of retrieved images \( N_r = 1, \ldots, N \) is computed as

\[
P = \frac{1}{NN_q} \sum_{i=1}^{N} \sum_{j=1}^{N_r} P_{i,j} \tag{9}
\]
where \( P_{i,j} \) indicates the precision value of the \( i \)\textsuperscript{th} query image after retrieving \( j \) images. The precision is defined as

\[
\text{Precision: } P_{i,j} = \frac{\text{Number of relevant images retrieved}}{\text{Total number of images retrieved}}.
\] (10)

We select one of the FEs which maximizes \( P \). Then we add another FE to our CBIR system. We select the second FE again for maximizing the value of \( P \). We continue the above process until the value of \( P \) starts to decrease. In each step, we check whether the computed \( P \) for the newly added FE is less than the precision value computed at the previous iteration, \( P_{\text{max}}^{(n-1)} \). If so, current FE is removed, since processing this FE at next steps cannot increase the precision.

This procedure ensures that the precision of final CBIR system using the selected FEs does not become less than that of the CBIR system using all FEs. Furthermore, using smaller feature vectors reduces the memory requirements of the image retrieval system and speeds up the retrieval process. Note that the proposed algorithm should be executed only once.

### 2.2.2 Calculation of Optimal Weights for Training

In this section, we present our approach for determining the function \( F \), defined in Section 2.2.1. This function is used to calculate weight vector \( \mathbf{W} \) from score vector \( \mathbf{S} \). This problem can be stated as follows: Find \( w_i \)s such that the following squared error minimizes:

\[
E = \sum_{n=1}^{N} \left[ D_{qt,n} - g^{-1}(s_n) \right]^2
\]

subject to:

\[
\sum_{i=1}^{M} w_i = 1
\] (11)

in which \( D_{qt,n} \) denotes the weighted distance between the query image and the \( n \)\textsuperscript{th} target image defined by Equation (1). \( g^{-1}(.) \) represents the inverse function defined in Equation (8). Incorporating the above constraint in the optimization procedure by using the Lagrange multiplier method, we obtain:

\[
\mathbf{W} = \mathbf{Y}^{-1} \left( \mathbf{X} - \frac{\mathbf{i}_M^T \mathbf{Y}^{-1} \mathbf{X} - 1}{\mathbf{i}_M^T \mathbf{Y}^{-1} \mathbf{i}_M} \mathbf{i}_M \right)
\] (12)

in which \( \mathbf{W} \) is an \( M \times 1 \) vector of elements \( w_j \), the weight of the \( j \)\textsuperscript{th} FE for the query image. \( \mathbf{i}_M \) is an \( M \times 1 \) vector of elements 1 and \( \mathbf{X} \) is an \( M \times 1 \) vector defined as

\[
\mathbf{X} \triangleq \mathbf{Z}^T \times g^{-1}(\mathbf{S})
\] (13)
where \( \mathbf{S} \) is an \( N \times 1 \) vector of scores assigned to the database images for the query image, and \( \mathbf{Z} \) is an \( N \times M \) matrix defined as follows:

\[
\mathbf{Z} \triangleq |i \times \mathbf{F}_q^T - \mathbf{F}_t^T|
\]

(14)

where \( \mathbf{F}_q \) is an \( M \times 1 \) vector of features of the query image and \( \mathbf{F}_t \) is an \( M \times N \) matrix of features of the database images. In the above equations, the superscript \( T \) denotes the transposition operation on the corresponding matrix or vector. In Equation (12), \( \mathbf{Y} \) is an \( M \times M \) matrix defined as:

\[
\mathbf{Y} \triangleq \mathbf{Z}^T \times \mathbf{Z}.
\]

(15)

Fig. 2. Fuzzy sets used in the fuzzy rules

The proof of Equation (12) is given in the Appendix. Finally, the function \( F \) is defined as \( F(\mathbf{S}) = \mathbf{W} \).

### 2.2.3 Determining the Premise and Consequent Fuzzy Set Parameters

As a result of fuzzy clustering, every output \( w_j \) is associated with the grade of membership belonging to a fuzzy cluster \( B_j \). Notice that we now have the following data associated with the grade of the membership of \( w_j \) in \( B_k \) (\( 1 \leq k \leq N_r \), \( 1 \leq j \leq M \)). We can induce a fuzzy cluster \( A \) in the input space. By making the projection of the cluster \( A \) onto the axes of the coordinates \( x_1 \) and \( x_2 \), we obtain the fuzzy sets \( A_1 \) and \( A_2 \). As it is easily seen, at this stage we have the following equation in which \( B \) is the output cluster.

\[
A_1(f_1) = A_2(f_2) = B_1(w_1) = B_2(w_2).
\]

(16)
Algorithm 3: Fuzzy partitioning of the input space.

input: The set of fuzzy clusters of the consequence fuzzy rules
output: The set of fuzzy clusters of the premise fuzzy rules

1 foreach cluster do
2 Compute average precision $P_1$ from Equation (9);
3 Find the optimum sub-clustering of the cluster using Equation (5);
4 Compute average precision $P_2$ from Equation (9) for these newly computed clusters;
5 if $P_2 > P_1$ then Split the cluster to these new optimal sub-clusters;
6 end

To calculate the trapezoidal-shaped fuzzy set parameters, we propose the following equations:

$$p_{2,i}^j = \min \left\{ f_i | A(F^q) = \max \{ A(F^q) | F^q \in C^j \} \right\}$$
$$p_{3,i}^j = \max \left\{ f_i | A(F^q) = \max \{ A(F^q) | F^q \in C^j \} \right\}$$
$$p_{1,i}^j = f_{\min,i}^j - A_i^j (f_{\min,i}^j) \frac{p_{2,i}^j - f_{\min,i}^j}{1 - A_i^j (f_{\min,i}^j)}$$
$$p_{4,i}^j = f_{\max,i}^j - A_i^j (f_{\max,i}^j) \frac{p_{3,i}^j - f_{\max,i}^j}{1 - A_i^j (f_{\max,i}^j)}$$  \hspace{1cm} (17)

where

$$f_{\min,i}^j = \min \left\{ f_i | F^q \in C^j \right\}$$
$$f_{\max,i}^j = \max \left\{ f_i | F^q \in C^j \right\}$$  \hspace{1cm} (18)

The parameter $p_{k,i}^j$ denotes the $k$th parameter of the $i$th fuzzy set in the $j$th rule depicted in Figure 2. In Equation (17), $C^j$ denotes the $j$th cluster, and $f_i$ is the $i$th element of the vector $F^q$. $p_{2,i}^j$ and $p_{3,i}^j$ are determined as the end points for which the membership value of corresponding fuzzy cluster maximizes. $p_{1,i}^j$ is the intersection of the line between membership values of $p_{2,i}^j$ and the member point with minimum value by the line $f_i = 0$. $p_{4,i}^j$ is computed in the same way. Note that this is an initial guess for the parameters $p_{k,i}^j$, whose value will be adjusted using an adjusting algorithm described in Section 2.2.4. Equation (17) is written for premise fuzzy set parameters; however, it can be used in the same way to find consequent fuzzy set parameters replacing $f_i$'s by $w_i$'s, and other parameters appropriately.

Now, this cluster gives a fuzzy rule: if $f_1$ is $A_1$ and $f_2$ is $A_2$, then $w_1$ is $B_1$ and $w_2$ is $B_2$.

Remark 1. Although the output cluster $B$ is convex, the input cluster $A$ corresponding to $B$ might not be convex. In this case, we approximate the input cluster
Algorithm 4: Adjusting the Premise Fuzzy Set Parameters.

**input**: The set of parameters of the premise fuzzy sets

**output**: The set of adjusted parameters of the premise fuzzy sets

1. $a \leftarrow$ an appropriate value;
2. // Suppose that the $k$th parameter of the $i$th fuzzy set in the $j$th fuzzy rule is $p_{k,i}^j$ (Figure 2)
3. **foreach** fuzzy rule $j$ **do**
4. **foreach** fuzzy set $i$ in the premise of fuzzy rule $i$ **do**
5. **foreach** parameter $k$ ($k = 1, 2, 3, 4$) **do**
6. $\hat{p} \leftarrow p_{k,i}^j + a$;
7. $\bar{p} \leftarrow p_{k,i}^j - a$;
8. if $k = 1, 2, 3$ and $\hat{p} > p_{k+1,i}^j$ then $\hat{p} \leftarrow p_{k+1,i}^j$;
9. if $k = 2, 3, 4$ and $\bar{p} < p_{k-1,i}^j$ then $\bar{p} \leftarrow p_{k-1,i}^j$;
10. Choose the parameter which shows the best performance $P$ in Equation (9) among $\{\hat{p}, p_{k,i}^j, \bar{p}\}$ and replace $p_{k,i}^j$ with it;
11. if $\hat{p}$ or $\bar{p}$ is chosen then
12. **repeat**
13. \hspace{1em} change $p_{k,i}^j$ in the same direction by $a$;
14. **until** $P$ decreases or $p_{k,i}^j$ reaches to its neighbor parameter;
15. **end**
16. **end**
17. **end**
18. **end**

With a convex fuzzy set. Finally, we approximate this convex fuzzy set and $B$ as well, with a fuzzy set of trapezoidal type as shown in Figure 2, which is used in the fuzzy model.

**Remark 2.** The next problem is that we might have more than two fuzzy clusters, $A_1$ and $A_2$ in the input space which corresponds to the same fuzzy cluster $B$ in the output space. In this case we carefully form two convex fuzzy clusters. We obtain the following two rules with the same consequent:

- $R^1$: if $f_1$ is $A_1^1$ and $F_2$ is $A_2^1$ then $w_1$ is $B_1$ and $w_2$ is $B_2$
- $R^2$: if $f_1$ is $A_1^2$ and $F_2$ is $A_2^2$ then $w_1$ is $B_1$ and $w_2$ is $B_2$.

As one can easily find, a fuzzy partition of the input space is obtained as a direct result of fuzzy clustering. Here, we propose the Algorithm 3.
2.2.4 Adjusting the Premise Fuzzy Set Parameters

In this section, we propose an algorithm for adjusting the fuzzy set parameters. This process leads to better precisions for the image retrieval task. Algorithm 4 is proposed for this purpose.

We use 2% of the width of the universe of discourse as the value of $a$. Note that we do not adjust the parameters in the consequents of the rules. This algorithm reaches to a local maximum of precision parameter $P$. This local maximum is sufficient for most of the situations.

3 EXPERIMENTAL RESULTS

In this section, we provide our experimental results on making a fuzzy system for semantic-based image retrieval. Our target database involves 59,600 images containing 596 folders from the Corel database and classified them into 174 classes. Our training database contains 2,000 images out of these 59,600 images. We assigned similarity scores to 200 query images selected randomly from this training set. We used a rule of thumb for determining the size of training set as follows: use about 10 images per each class (in our experiments: 2,000 images). This is an empirical rule which is proved to provide sufficient training data in our experiments. If we have 10 million images containing 100 classes, only 1,000 images are sufficient for training the system (i.e., about 0.01 of total images).

3.1 Low Level Features

Two features were extracted from the images discussed in the following sections.

3.1.1 Texture

We used the Pattern Orientation Histogram (POH) method as the texture features which is based on the pattern orientations in spatial domain [25]. POH represents distribution of five types of patterns from each image and produces 80 bins histogram. Increasing the size of image blocks in the POH method results in an increase of the precision of the image retrieval system up to an optimal value from which the precision deteriorates by more increasing the image blocks. This optimal value depends on the database images. We used a moderate value which is near to the optimal value computed for our database. However, this is not the concern of this paper, since we do not focus on the low-level features, but our main concern is to enhance the performance of the CBIR systems with any low-level feature used in them.

3.1.2 Color

A good color space is one in which the perceived color differences should correspond to their Euclidean distances in this chosen color space. The HSV color model is
known to satisfy this property. We quantize the HSV color space into 162-bin color histogram. These values are achieved by a uniform quantization, which includes 18 levels in $H$, three levels in $S$, and three levels in $V$ color space.

### 3.2 Results of Running Feature Selection Algorithm

After running our algorithm, 28 FEs among 242 FEs are selected. Figure 3 depicts the average precision value obtained using Equation (9) for different number of selected FEs. As shown in the figure, the dashed line indicates the average precision value for the case of using all FEs in the retrieval process. This line is used for comparison with the reduced dimensionality cases. It is worthwhile to say that the average precision value for the case of ideal image retrieval in our experiment is 0.2262. This is due to the computation of precision for the number of retrieved images from 1 to $N$, and it is clear that in each query, some images are irrelevant and computing precision for $N$ returned images, even in the case of ideal image retrieval, results in values less than one.

![Average precision values vs. the number of FEs using PCA and our proposed feature dimensionality reduction method](image)

Fig. 3. Average precision values vs. the number of FEs using PCA and our proposed feature dimensionality reduction method

One of the most popular approaches in reduction of feature space dimensionality is to use the principle component analysis (PCA) approach. In Figure 3, the dashed line represents the average precision value for different number of FEs from the PCA representation space of each image. As can be seen, using PCA reduces the average precision of the image retrieval process with respect to the case of using all FEs in the histogram space. Note that the average precision value maximizes at 7 FEs for this case. Finally, the line with “+” signs indicates the average precision resulted from running our algorithm for finding appropriate FEs. Clearly, the curve reaches to its maximum at 49 elements. We used the first 28 elements in our experiments, since the variations from 28 to 49 elements are negligible.
Fig. 4. Results of Query Example 1: Retrieval results for different methods; a) CBIR; b) Classification; c) MindReader; d) FRBF; e) FRC; f) IRFuM (Images order: row-wise from left to right, i.e. the first row is returned first, then the second row and so on; note that some images are rotated by 90° counter-clockwise to fit in the table)
3.3 Results of Image Retrieval System

3.3.1 Query Example 1

Figure 4 shows the retrieval results for a sample query image using simple CBIR method, a typical classification-based image retrieval methods, the MindReader [26], the FRBF [7], the FRC [6] and our proposed method (IRFuM). To have a reasonable comparison, we used the same training set for making the weight matrix of the MindReader method and as the feedbacks for FRBF and for the fuzzy classification of the FRC. Thus, we do not make relevance feedback, and use the training set instead.

Figure 4 a) shows the retrieval results for the CBIR. In this method, the $L^1$ distance measure is used for determination of the distance between image feature vectors, and images with less distance values are retrieved first. The error of system due to semantic gap is evident in this figure, since only a few relevant images are returned.

Figure 4 b) depicts the first 48 images retrieved using a typical classification method. In this method, the same training set and the same number of classes are used for the classification. The minimum distance measure is used for decision rule and then images are labeled by the predefined labels. We tried to make similar conditions for fuzzy modeling and image classification to enable to compare these methods. We emphasize on this fact that fuzzy modeling is a semantic layer over a CBIR layer and it can be made independent of its CBIR layer properties such as the features. As can be seen in Figure 4 b), the errors of classification affects the retrieval results. The retrieved images are extracted from a class made in this system with less distance to the query image. As can be seen, the system could not classify the images in a perfect manner and irrelevant images are put in one class. One major source of such errors is the diversity of database images. If we restrict the database to have a limited number of classes, the precision will be arisen. However, the results are more precise than the simple CBIR method.

Figure 4 c) shows the retrieved images for the MindReader method. In this case, the number of relevant images is more than the previous methods. The MindReader also tries to adjust the feature weights by using the similarities. However, since the irrelevant images are omitted from the computations of this method (because of the zero score given for them), our method is expected to provide more precise results.

In Figure 4 d), the retrieved images for the FRBF method are shown. FRBF is a fuzzy relevance feedback method which uses RBF neural network to learn the feedbacks given by the users. More relevant images are retrieved with respect to the MindReader in this example. This shows the superiority of the RBF to learn the user feedbacks with respect to movement of the query image used by the MindReader.

Figure 4 e) depicts the retrieved images using FRC, another image retrieval method which uses fuzzy logic. FRC classifies the images into fuzzy classes in the color space. Although this method is more precise than the CBIR, but it produces more irrelevant images with respect to the MindReader and FRBF methods. The
main sources of the FRC errors are the classification errors and representation of the images just by the color features.

Finally, Figure 4f) shows the retrieval results for our proposed fuzzy modeling method, IRFuM. As can be seen, more relevant images are returned by our method which shows the reduction of semantic gap in it. Using different weights for different FEs resulted in the retrieval of some images with equivalent semantic meanings but with far feature vectors. Obviously, using fuzzy modeling leads to retrieval of more relevant images. However, some irrelevant images are observed which is due to using low level features. One can decrease these errors by using more sophisticated feature elements. Note that the proposed method can be used beside other semantic based image retrieval systems to achieve a system with higher semantic capabilities.

Another source of error in our experiments was the large size of target database and the wide diversity of the images, i.e., we have too many types of images in our database which affects the retrieval precision. Additionally, if we use larger sets for the training, the retrieval performance will be enhanced.

3.3.2 Query Example 2

Figure 5 shows another example for retrieval of images by different methods. In this example, the FRC is less precise with respect to the simple classification method. The source of such errors is mainly due to inefficient detection of appropriate fuzzy class. Again, our proposed system returned more relevant images with respect to other ones.

3.4 Precision-Recall Plots

Figure 6 shows the precision-recall plots for some sample query images. The recall criterion is defined as follows:

\[
\text{Recall} = \frac{\text{Number of relevant images retrieved}}{\text{Total number of relevant images}}.
\] (19)

In these plots, the dashed green lines indicate the precision-recall values for the simple CBIR method. The dashed-dotted red lines represent the performance indices for the classification method. The dotted blue lines show the performance indices for the MindReader method. The FRBF and FRC methods are given by the dashed magenta and dashed-dotted yellow curves, respectively. Finally, the solid black and solid cyan lines represent IRFuM and the ideal cases, respectively. From these plots, some important points are concluded:

- In almost all cases, IRFuM is the most precise and CBIR is the most imprecise method.
- Comparing FRC with classification method, we can conclude that in most cases the FRC method outperforms the simple classification method. This shows that fuzzy classification is better than crisp classes. However, in some cases such as
Fig. 5. Results of Query Example 2: Retrieval results for different methods; a) CBIR; b) Classification; c) MindReader; d) FRBF; e) FRC; f) IRFuM
Figures 6 a), j), k), l), n), the situation reverses. These are the cases in which the correct class is not recognized for the given query by the FRC method.

- Comparing classification with the MindReader method, one can realize that MindReader is more successful than the classification method in most cases which shows the success of the idea of weighting the feature elements. However, in some cases such as Figures 6 i), l), the classification method outperforms the MindReader which is due to omission of irrelevant images in its computations.
- FRBF works better than MindReader in most cases which shows that using neural RBF networks specially by the fuzzy feedbacks can enhance the performance of the image retrieval systems. However, in some cases, such as Figures 6 j), n), the FRBF failed to produce relevant images. The source of such errors was mainly due to inadequate training of the RBF network.

The content of Table 3 consists of the precision and recall values of the first 9 query images of Figure 6 for the first 11, 21, ..., and 81 retrieved images in
Fig. 6. Precision-recall plots for some sample query images: - - (green): CBIR, - - - (red): Classification, . . . (blue): MindReader, -- (magenta) FRBF, -- (yellow): FRC, — (black): IRFuM, — (cyan): Ideal

each method. Each row contains the results for simple CBIR method (CBIR), the classification system (Class), the MindReader method (MR), FRBF, FRC, our proposed method (IRFuM), and the ideal case. Based on the content of these tables, the superiority of the proposed system over other methods can be concluded again.

Figure 7 shows the average precision-recall values measured for 50 different query images. The average precision-recall values of the ideal system is also depicted in this figure. Since the number of relevant images differs for different query images and for any query image, most of the images are irrelevant, retrieving all images results in precision values of less than one in ideal case. As shown, our proposed fuzzy system results in better average precision-recall values compared to other approaches. However, the distance to the ideal system is still far which is due to the fact that we yet use the low level features as the basis for comparing images in our system. As mentioned before, for improving the performance of the system, these solutions can be advantageous:
<table>
<thead>
<tr>
<th>Method</th>
<th>Precision</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBIR</td>
<td>0.818</td>
<td>0.1429</td>
</tr>
<tr>
<td>Class</td>
<td>0.545</td>
<td>0.3810</td>
</tr>
<tr>
<td>MR</td>
<td>0.7275</td>
<td>0.4286</td>
</tr>
<tr>
<td>FRBh</td>
<td>0.2725</td>
<td>0.2362</td>
</tr>
<tr>
<td>FRC</td>
<td>0.545</td>
<td>0.2877</td>
</tr>
<tr>
<td>IRFuM</td>
<td>0.8182</td>
<td>0.4286</td>
</tr>
<tr>
<td>Ideal</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

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<tr>
<th>Method</th>
<th>Precision</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBIR</td>
<td>0.9909</td>
<td>0.0476</td>
</tr>
<tr>
<td>Class</td>
<td>0.8184</td>
<td>0.1572</td>
</tr>
<tr>
<td>MR</td>
<td>0.4455</td>
<td>0.2381</td>
</tr>
<tr>
<td>FRBh</td>
<td>0.1819</td>
<td>0.1613</td>
</tr>
<tr>
<td>FRC</td>
<td>0.2727</td>
<td>0.1905</td>
</tr>
<tr>
<td>IRFuM</td>
<td>0.4455</td>
<td>0.2381</td>
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<tr>
<td>Ideal</td>
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</tr>
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</tr>
<tr>
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<td>0.4455</td>
<td>0.2381</td>
</tr>
<tr>
<td>Ideal</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 3. Precisions and recalls computed at different number of retrieved images for each method.
Fig. 7. Average precision-recall computed for different methods over 50 query images

1. Using more sophisticated image features by which the relevant and irrelevant points became far from each other.
2. Restricting the image database to some certain images and reducing the diversity of the images.
3. Using more bigger training set for making the fuzzy rules.

Our main purpose in this paper is to emphasize that fuzzy modeling has the potential for reduction of the semantic gap in CBIR systems, which is evident from our experimental results.

<table>
<thead>
<tr>
<th>Training</th>
<th>Addition and Multiplication</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBIR</td>
<td>(O(0))</td>
<td>(O(0))</td>
</tr>
<tr>
<td>Class</td>
<td>(O(\mathcal{F}M))</td>
<td>(O(0))</td>
</tr>
<tr>
<td>MR</td>
<td>(O(0))</td>
<td>(O(0))</td>
</tr>
<tr>
<td>FRBF</td>
<td>(O(0))</td>
<td>(O(0))</td>
</tr>
<tr>
<td>FRC</td>
<td>(O(\mathcal{F}M))</td>
<td>(O(0))</td>
</tr>
<tr>
<td>IRFuM</td>
<td>(O(M^3F) + O(m^2) + O(cFm) + O(mN_r) + O(N_r m^2F))</td>
<td>(O(M^3F) + O(cFm) + O(N_r m^2F))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Runtime</th>
<th>Addition and Multiplication</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBIR</td>
<td>(O(MN))</td>
<td>(O(RN))</td>
</tr>
<tr>
<td>Class</td>
<td>(O(MC))</td>
<td>(O(RC))</td>
</tr>
<tr>
<td>MR</td>
<td>(O(MN_F) + O(M^2N_F) + O(M^2) + O(M^3))</td>
<td>(O(RN))</td>
</tr>
<tr>
<td>FRBF</td>
<td>(O(M^3F) + O(WM_F))</td>
<td>(O(RN))</td>
</tr>
<tr>
<td>FRC</td>
<td>(O(MC))</td>
<td>(O(RC))</td>
</tr>
<tr>
<td>IRFuM</td>
<td>(O(mN_r) + O(mN))</td>
<td>(O(mN_r) + O(RN))</td>
</tr>
</tbody>
</table>

Table 4. The order of computations for different methods
3.5 Comparison of the Methods Based on Their Computational Complexity

To compare the computational complexity of the methods introduced in our experiments (Section 3.3), we use the $O$-notation to show the order of computations based on the relevant variables. We decompose the computations into two parts: training and run-time. The training part denotes the training phase of each method which is done offline. On the other hand, the runtime part is done when a query is given to the system and the computations are done online. Table 4 shows the orders of computations for each method.

The meanings of the variables are given in Table 1. In this table, the “+” sign is used to indicate different parts of computations for each method. For example, at the training, the values given for IRFuM multiplications and additions come from dimensionality reduction, computation of $W$, optimum clustering, initial value for consequent and premise fuzzy rules parameters, and adjusting the premise fuzzy set parameters, respectively. In the same column, the runtime values for the MR are coming from the computation of the query vector, covariance matrix, weight matrix, and generalized Euclidean distance. The corresponding values for the FRBF are coming from the computations for query and training the RBF network. The computations given for the IRFuM at runtime are coming from the search for the rules and database images. Note that these computations can be done faster, if we use more sophisticated methods; but, we just use this table as a rule of thumb for the comparison.

The CBIR, MR, and FRBF methods have no training phases, and so their computational complexity has $O(0)$. Note that, the MR and FRBF methods learn the user feedbacks on runtime and have no offline training. The FRC and our defined simple classification method have the same order, both for the training and runtime, but with different values of $M$ and $C$. However, the exact numbers of computations differ in these methods, since they use different algorithms. However, they have the same order for their computations. These methods has the least computations among the six compared methods at runtime, since the product of $MC$ is typically less than $mN$ (IRFuM). The computations of the IRFuM lie after these classification based methods at runtime. Depending on the number of database images and user feedbacks, the worst case arises from FRBF or MR methods at runtime. On the other hand, the IRFuM has more computations at training phase which reflects that we pass most computation burdens offline to have a relatively fast runtime part and to achieve a precise image retrieval system, comparing these methods.

4 CONCLUSION

We designed a fuzzy system to reduce the semantic gap of the current content-based image retrieval approaches. Our main contribution was in designing the fuzzy modeling itself, the structure for semantic-based image retrieval via a fuzzy system, and the training algorithms for making different parts of the fuzzy system. We deve-
developed a method for converting scores of similarity between query and target images to output data needed in the training phase. Our experiments on a dataset of 59,600 images from the Corel database show that using fuzzy system for adjusting weights for feature elements in the distance measure can improve the precision-recall performance of the CBIR system. Moreover, we have shown that our system outperforms the crisp and a fuzzy classification-based image retrieval system (FRC), the feature weighting method called MindReader, a fuzzy relevance feedback method using RBF neural network (FRBF) and the simple CBIR method from the precision-recall point of view. Comparing these methods based on their computation complexities, we arrive at the point that the proposed method achieves its slightly better precision with respect to the FRBF and MindReader and its apparent superiority over other mentioned methods by making more computations at training phase which is done offline. Moreover, the proposed method makes less computations at runtime with respect to the MindReader and FRBF methods, transferring some computation burdens to the offline training phase.

APPENDIX

A DERIVATION OF OPTIMAL WEIGHTS

By using the Lagrange multiplier method, this constrained minimization problem can be converted to minimization of

\[ J = \sum_{n=1}^{N} \left[ \left( \sum_{i=1}^{M} w_i |f_q^i - f_t^i| \right) - g^{-1}(s_n) \right]^2 + \lambda \left( \sum_{i=1}^{M} w_i - 1 \right) \]  \hspace{1cm} (20)

where \( \lambda \) must be chosen to satisfy Equation (12). In Equation (20), \( g^{-1}(.) \) is a function for converting scores to distances and is defined as the inverse function defined in Equation (8) and equals to:

\[ g^{-1}(s) = \frac{1}{5}(5 - s) \]  \hspace{1cm} (21)

in our experiments. Let \( \partial J/\partial w_k = 0 \); then

\[ 2 \sum_{n=1}^{N} \left[ \left| f_k^q - f_k^t \right| \left( g^{-1}(s_n) - \sum_{i=1}^{M} w_i |f_q^i - f_t^i| \right) \right] = \lambda. \]  \hspace{1cm} (22)

Using Equations (13)–(15), Equation (22) reduces to

\[ \mathbf{X} - \mathbf{Y} \times \mathbf{W} = \frac{\lambda}{2} i_M. \]  \hspace{1cm} (23)
Multiplying both sides of Equation (23) by $i_M^T Y^{-1}$ and by considering that $i_M^T W = \sum_{i=1}^{M} w_i = 1$, we get

$$i_M^T Y^{-1} X - 1 = \frac{\lambda}{2} i_M^T Y^{-1} i_M. \quad (24)$$

Solving Equation (24) for $\lambda$ and substituting the results in Equation (23), we obtain

$$X - Y W = - \left( \frac{i_M^T Y^{-1} X - 1}{i_M^T Y^{-1} i_M} \right) i_M \quad (25)$$

Equation (12) is calculated by multiplying both sides of Equation (25) by $Y^{-1}$ and solving the equation for $W$.

REFERENCES


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